O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

ISLOM KARIMOV nomidagi TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

Oliy matematika

sirtqi ta’lim talabalari uchun uslubiy ko‘rsatmalar

1-qism

Toshkent–2019

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Ushbu uslubiy ko'rsatmalar "Oliy matematika" fanining matritsalar algebrasi, chiziqli tenglamalar sistemasi va determinantlar nazariyasi, analitik geometriya, matematik analizga kirish bo'limlarini o'z ichiga olgan.

Mazkur uslubiy ko'rsatma oliy texnika o'quv yurtlari sirtqi ta'lim yo'nalishlari uchun tasdiqlangan o'quv rejasi asosida tayyorlangan.

Uslubiy ko'rsatmada matritsalar ustida amallar, determinantlarni hisoblash, bir jinsli va bir jinsli bo'lmagan tenglamalar sistemasini yechishning asosiy usullari, vektorlar tushunchasi, ikki vektorning skalyar, vektor va aralash ko'paytmalari, tekislikda to'g'ri chiziqlar, tekislik, fazoda to'g'ri chiziqlar va to'g'ri chiziq va tekislik orasidagi munosabatlar keltirilgan.

Uslubiy ko'rsatmalar, universitetning barcha mutaxassislikdagi sirtqi ta'lim 1-bosqich talabalari foydalanishlari uchun mo'ljallab tayyorlangan.

Har bir bo'limda shu bo'limga tegishli nazariyaning asosiy tushunchalari va formulalari keltirilgan bo'lib, har biri 30 ta variantdan iborat bo'lgan 10 ta mustaqil ish topshiriqlari berilgan.

Har bir topshiriq uchun namunaviy variant olinib, ya'ni shu topshiriqqa mos misollar olinib ularning to'liq yechiimlari keltirilgan.

Uslubiy ko'rsatmalar Islom Karimov nomidagi Toshkent davlat texnika universiteti ilmiy-uslubiy kengashi tomonidan nashrga tavsiya etilgan.

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I. Chiziqli algebra

1.1. Matritsa va ular ustida amallar

Matritsa deb sonlarning to‘g‘ri burchakli jadvaliga aytiladi.

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

Belgilashlar: A- matritsa; a_{ij} - matritsa elementlari; i - berilgan element joylashgan satr raqami; j - unga mos ustun raqami; m - matritsadagi satrlar soni; n- undagi ustunlar soni.

Agar $m=n$ bo‘lsa matritsa kvadrat matritsa deb ataladi. n- soni matritsaning tartibi deyiladi.

Bir xil o‘lchamga ega bo‘lgan matritsalarning mos elementlari o‘zaro teng bo‘lsa, bunday matritsalar o‘zaro teng matritsalar deb ataladi.

Agar matritsaning barcha elementlari nollardan iborat bo‘lsa, bunday matritsa nolli matritsa deb ataladi.

Agar kvadrat matritsaning asosiy diagonalidagi barcha elementlari 1, qolganlari 0 bo‘lsa, bunday matritsa birlik matritsa deb ataladi.

Matritsalar ustida chiziqli amallar

Matritsalarni qo‘shish

Bir hil $m\times n$ o‘lchamli A va B matritsalarning yig‘indisi deb, huddi Shunday o‘lchamli C matritsaga aytiladiki, bu matritsaning har bir elementi A va B matritsalarning mos elementlarining yig‘indisidan iborat bo‘ladi:

$$c_{ij}=a_{ij}+b_{ij},\ i=\overline{1,m},\ j=\overline{1,n}$$

1-misol. $A=\begin{pmatrix}2 & -3 & 1 & 1\\ 0 & 4 & -2 & 8\end{pmatrix}$ va $B=\begin{pmatrix}-1 & 4 & 0 & -1\\ 2 & -2 & 5 & 7\end{pmatrix}$ matritsalarning yig‘indisini toping.

Yechish. Berilgan matritsalarning bir xil joyda turgan elementlarini qo‘shib, $C = A + B$ matritsa elementlarini hisoblaymiz.

$c_{11}=a_{11}+b_{11}=2-1=1;\ c_{12}=-3+4=1;\ c_{13}=1+0=1;\ c_{14}=1-1=0;$

$c_{21}=0+2=2;\ c_{22}=4-2=2;\ c_{32}=-2+5=3;\ c_{14}=8+7=15.$

Shunday qilib, C $=A+B=\begin{pmatrix}1 & 1 & 1 & 0\\ 2 & 2 & 3 & 15\end{pmatrix}.$

Matritsani songa ko‘paytirish

Matritsani songa ko‘paytirish deb, o‘lchami berilgan matritsa o‘lchamiga teng bo‘lgan, har bir elementi berilgan matritsa elementini berilgan songa ko‘paytirishdan hosil bo‘lgan matritsaga aytiladi.

2-misol. Agar $A=\begin{pmatrix}2 & -3 & 1\\ -1 & 0 & -2\end{pmatrix},\ B=\begin{pmatrix}4 & 3 & 2\\ -3 & 1 & -4\end{pmatrix}$ bo‘lsa, 5A – 2B matritsani toping.

Yechish. $5A=\begin{pmatrix}10 & -15 & 5\\ -5 & 0 & -10\end{pmatrix},\ 2B=\begin{pmatrix}8 & 6 & 4\\ -6 & 2 & -8\end{pmatrix},$

$$5A-2B=\begin{pmatrix}10-8 & -15-6 & 5-4\\ -5+6 & 0-2 & -10+8\end{pmatrix}=\begin{pmatrix}2 & -21 & 1\\ 1 & -2 & -2\end{pmatrix}$$

Shunday qilib, $5A-2B=\begin{pmatrix}2 & -21 & 1\\ 1 & -2 & -2\end{pmatrix}.$

Matritsalarni ko‘paytirish

O‘lchami $m\times p$ bo‘lgan A matritsa va ulchami $p\times n$ bo‘lgan B matritsalarning ko‘paytmasi deb o‘lchami $m\times n$ bo‘lgan Shunday C matritsaga aytiladiki, uning har bir elementi c_{ij} quyidagi formula bilan aniqlanadi: $c_{ij}=\sum_{k=1}^{p}a_{ik}b_{kj},\quad i=1,...,m,\ j=1,...,n.$

Shunday qilib, c_{ij} element A matritsaning *i* - satrini B matritsaning unga mos *j* - ustuniga ko‘paytmasining yig‘indisidan iborat ekan.

Matritsalarni ko‘paytirish amali kommutativ emas, ya’ni $AB \neq BA$. Haqiqatdan ham, AB ko‘paytma mavjud bo‘lsa, o‘lchamlari to‘g‘ri kelmasligi sababli BA ko‘paytma umuman mavjud bo‘lmasligi mumkin. Agar AB va BA lar mavjud bo‘lsa ham, ularning o‘lchamlari har hil bo‘lishi mumkin.

Bir hil o‘lchamli kvadrat matritsalar uchun AB va BA ko‘paytmalar mavjud va ular bir hil o‘lchamga ega bo‘ladi, ammo umuman olganda mos elementlari teng bo‘lmaydi.

3-misol. Quyidagi matritsalarni bir-biriga ko‘paytirish mumkinmi yoki yo‘qmi? Shuni aniqlang. Agar ko‘paytma mavjud bo‘lsa, uni hisoblang.

$A = \begin{pmatrix} 0 & 3 \\ 4 & -2 \\ 1 & -1 \end{pmatrix}$ va $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$.

Yechish. A va B matritsalarning o‘lchamlarini taqqoslaymiz. $A[3\times 2]$, $B[2\times 2]$. Bundan $n = l$, $m \neq k$, shuning uchun $AB[3\times 2]$ mavjud, ko‘paytma BA esa mavjud emas.

AB ko‘paytma elementlarini topamiz:

$(ab)_{11} = 0 \cdot 5 + 3 \cdot 7 = 21$; $(ab)_{12} = 0 \cdot 6 + 3 \cdot 8 = 24$; $(ab)_{21} = 4 \cdot 5 – 2 \cdot 7 = 6$;

$(ab)_{22} = 4 \cdot 6 – 2 \cdot 8 = 8$; $(ab)_{31} = 1 \cdot 5 – 1 \cdot 7 = -2$; $(ab)_{32} = 1 \cdot 6 – 1 \cdot 8 = -2$.

Shundayqilib, $AB = \begin{pmatrix} 21 & 24 \\ 6 & 8 \\ -2 & -2 \end{pmatrix}$, BA mavjud emas.

4-misol. Agar $A = \begin{pmatrix} 2 & -2 & 1 & 0 \\ -3 & 1 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ -1 & 0 \\ 1 & 1 \\ 2 & 4 \end{pmatrix}$ bo‘lsa, AB va BA ni toping.

Yechish. Matritsalarni ko‘paytirish mumkinmi yoki yo‘qligini bilish uchun ularning o‘lchamlarini aniqlaymiz.

$A[2\times4]$, $B[4\times2]$. Bundan $n = l = 4$, $m = k = 2$, shuning uchun AB va BA matritsalar mavjud, hamda $AB[2\times2]$, $BA[4\times4]$.

$C = AB$ matritsaning elementlarini hisoblash uchun$A$ matritsaning satr elementlarini unga mos bulgan B matritsaning ustun elementlariga ko‘paytiriladi.

$c_{11} = 2 \cdot 2 + (-2)(-1) + 1 \cdot 1 + 0 \cdot 2 = 9$

(A ning birinchi satr elementlarining B ning birinchi ustun elementlariga Ko‘paytmasining yig‘indisi; hisoblanayotgan elementning birinchi indeksi A matritsaning satrini, ikkinchi indeksi esa B matritsa ustunini bildiradi).

$c_{12} = 2 \cdot 2 + (-2) \cdot 0 + 1 \cdot 1 + 0 \cdot 4 = 5;$

$c_{21} = -3 \cdot 3 + 1 \cdot (-1) + (-1) \cdot 1 + 1 \cdot 2 = -9;$

$c_{22} = -3 \cdot 2 + 1 \cdot 0 + (-1\_ \cdot 1 + 1 \cdot 4 = -3.$

Shunday qilib, $C = AB = \begin{pmatrix} 9 & 5 \\ -9 & -3 \end{pmatrix}$.

$D = BA$ matritsa elementlarini hisoblayotganda B ning satr elementlari A ning ustun elementlariga ko‘paytiriladi.

$d_{11} = 3 \cdot 2 + 2 \cdot (-3) = 0;$ $d_{12} = 3 \cdot (-2) + 2 \cdot 1 = -4;$ $d_{13} = 3 \cdot 1 + 2 \cdot (-1) = 1;$
$d_{14} = 3 \cdot 0 + 2 \cdot 1 = 2;$ $d_{21} = -1 \cdot 2 + 0 \cdot (-3) = -2;$ $d_{22} = -1 \cdot (-2) + 0 \cdot 1 = 2;$
$d_{23} = -1 \cdot 1 + 0 \cdot (-1) = -1;$ $d_{24} = -1 \cdot 0 + 0 \cdot 1 = 0;$ $d_{31} = 1 \cdot 2 + 1 \cdot (-3) = -1;$
$d_{32} = 1 \cdot (-2) + 1 \cdot 1 = -1;$ $d_{33} = 1 \cdot 1 + 1 \cdot (-1) = 0;$ $d_{34} = 1 \cdot 0 + 1 \cdot 1 = 1;$
$d_{41} = 2 \cdot 2 + 4 \cdot (-3) = -8;$ $d_{42} = 2 \cdot (-2) + 4 \cdot 1 = 0;$ $d_{43} = 2 \cdot 1 + 4 \cdot (-1) = -2;$
$d_{44} = 2 \cdot 0 + 4 \cdot 1 = 4.$

Shunday qilib, $D = BA = \begin{pmatrix} 0 & -4 & 1 & 2 \\ -2 & 2 & -1 & 0 \\ -1 & -1 & 0 & 1 \\ -8 & 0 & -2 & 4 \end{pmatrix}.$

1.2. Determinantlar

Iikinchi tartibli determinant deb, ikkinchi tartibli kvadrat matritsa elementlari yordamida aniqlanuvchi quyidagi songa aytiladi.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

Determinantning bosh diagonalida joylashgan elementlar ko‘paytmasidan, yordamchi diagonalda joylashgan elementlar ko‘paytmasi ayiriladi.

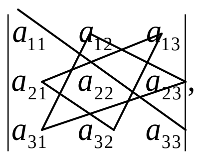
5-misol. $\begin{vmatrix} 1 & -3 \\ 5 & 8 \end{vmatrix} = 1 \cdot 8 - 5 \cdot (-3) = 8 + 15 = 23.$

Uchinchi tartibli determinant deb, uchinchi tartibli kvadrat matritsa elementlari yordamida quyidagicha aniqlanuvchi songa aytiladi.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

Bu formulani eslab qolish uchun uchburchaklar qoidasidan foydalanish mumkin. U quyidagilardan iborat:

ko‘paytmasi determinantga «+» belgisi bilan kiruvchi elementlar quyidagicha joylashadi:



Bosh diagonalga simmetrik bo‘lgan ikkta uchburchak hosil qilinadi. Ko‘paytmasi determinantga «-» belgisi bilan kiruvchi elementlar ham, huddi shu kabi, yordamchi diagonalga nisbatan joylashadi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

6-misol. Determinantni hisoblang.

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 0 & -4 \\ 2 & 1 & -1 \end{vmatrix}$$

Yechish. 3-chi tartibli determinantni uning qoidasidan foydalanib hisoblaymiz.

$$\Delta = 2\cdot0\cdot(-1) + (-3)\cdot(-4)\cdot2 + 5\cdot1\cdot1 - 2\cdot0\cdot5 - 1\cdot(-4)\cdot2 - (-1)\cdot1\cdot(-3) =$$

$$= 0 + 24 + 5 - 0 + 8 - 3 = 34.$$

Determinantlarning asosiy xossalarini berishdan oldin transponirlangan matritsa tushunchasining ta’rifini keltiramiz.

Transponirlangan matritsa deb, ularning joylanishlari saqlangan holda satr va ustunlarini almashtirishdan hosil bo‘lgan matritsaga aytiladi. Natijada A matritsaga nisbatan transponirlangan A' matritsa hosil bo‘ladi, uning elementlari A matritsa elementlari bilan quyidagi munosabatda bog‘lanadi: $a'_{ij} = a_{ji}$.

Determinantlarning asosiy xossalari

1. Transponirlash natijasida determinant o‘zgarmaydi, ya’ni

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}.$$

2. Determinantning satr(yoki ustun) elementlari biror songa ko‘paytirilsa, determinantning qiymati shu songa ko‘paytiriladi, ya’ni

$$\begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

3. Nolli satr(yoki ustun)ga ega bo‘lgan determinant nolga teng

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.$$

4. Ikkita bir hil satr(yoki ustun)ga ega bo‘lgan determinant nolga teng

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.$$

5. Ikkita satr(yoki ustun)i o‘zaro proporsional bo‘lgan determinant nolga teng

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.$$

6. Determinantda ikkita satr(yoki ustun)i o‘zaro almashtirilsa, uning qiymati (-1)ga ko‘paytiriladi.

$$\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

7. $\begin{vmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} b_1 & b_2 & b_3 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} c_1 & c_2 & c_3 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$

8. Determinantning biror satr(yoki ustun) elementlarini biror songa ko‘paytirib, ikkinchi satr(yoki ustun)ning mos elementlariga qo‘shilsa, determinantning qiymati o‘zgarmaydi.

$$\begin{vmatrix} a_{11}+ka_{21} & a_{12}+ka_{22} & a_{13}+ka_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

Determinatni satr yoki ustun bo‘yicha yoyish

Determinantning biror elementining **minori** deb, shu element turgan satr va ustunni o‘chirishdan hosil bo‘lgan determinantga aytiladi va M_{ij} bilan belgilanadi.

7- misol. $\begin{vmatrix} 1 & 2 & 3 \\ -5 & 1 & 1 \\ 2 & -1 & 4 \end{vmatrix}$ uchun $a_{21} = -5, \quad M_{21} = \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} = 8 + 3 = 11.$

Determinantning a_{ji} elementining algebraik to‘ldiruvchisi deb Shunday minorga aytiladiki, agar $i + j$ juft bo‘lsa, u minorning o‘ziga teng, $i + j$ toq bo‘lsa, minorga qarama-qarshi bo‘lgan songa teng, ya’ni $A_{ij} = (-1)^{i+j} M_{ij}$.

Shu bilan birga quyidagi tasdiq o‘rinlidir: Determinatning qiymati uning ihtiyoriy satr yoki ustun elementlarining ularga mos algebraik to‘ldiruvchilarga ko‘paytmasining yig‘indisiga teng, ya’ni

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \sum_{j=1}^{3} a_{ij} A_{ij,}$$ bu erda i=1,2,3.

Shunday qilib, determinatni hisoblash uchun qandaydir ustun yoki satr elementlarining algebraik to‘ldiruvchilarini topib, ularni determinantning mos elementlariga ko‘paytmasining yig‘indisini hisoblash etarlidir.

8-misol. 6 misoldagi determinantni satrga yoyish yordamida hisoblaymiz. Qulaylik uchun 2-satrni tanlaymiz, chunki a_{22} = 0 bo‘lganligidan $a_{22} \cdot A_{22} = 0$.

Shunday qilib,

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 5 \\ 1 & -1 \end{vmatrix} = -1 \cdot (-3 \cdot (-1) - 5 \cdot 1) = 2;$$

$$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix} = -1 \cdot (2 \cdot 1 - (-3) \cdot 2) = -8$$

U holda $\Delta = a_{21}A_{21} + a_{23}A_{23} = 1 \cdot 2 + (-4)(-8) = 34$.

Yuqori tartibli determinantlar

n - tartibli determinant deb n – ta satr va n – ta ustundan iborat bo‘lgan quyidagi determinantga aytiladi.

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Uchinchi tartibli determinantning barcha xossalari n- tartibli determinant uchun ham o‘rinlidir.

Amaliyotda yuqori tartibli determinantlarni satr yoki ustun bo‘yicha yoyishdan foydalanib xisoblanadi. Ustun yoki satr bo‘yicha yoyish natijasida determinantning tartibi pasaytiriladi va natijada uni uchinchi tartibli determinantga olib kelish mumkin.

9-Misol. 4- tartibli determinantni hisoblang.

$$\Delta = \begin{vmatrix} 2 & -5 & 1 & 2 \\ -3 & 7 & -1 & 4 \\ 5 & -9 & 2 & 7 \\ 4 & -6 & 1 & 2 \end{vmatrix}$$

Yechish. Determinantni Shunday almashtiramizki, natijada bir ustun yoki satrda to‘rtta elementdan uchtasi nolga aylansin. Buning uchun 8-xossadan foydalanamiz. Agar determinantda ± 1ga teng element bo‘lsa, bu xossani qo‘llash juda o‘rinli bo‘ladi. Shunday element sifatida $a_{13} = 1$ elementni tanlaymiz va uning yordamida 3- ustunning qolgan barcha elementlarini nolga aylantiramiz.

Shu maqsadda:

a) 2- satr elementlariga ularga mos 1- satr elementlarini qo‘shamiz;

b) 1- star elementlarini 2 ga ko‘paytirib 3- satr elementlaridan ayiramiz.

v) 4-satr elementlaridan 1-satr elementlarini ayiramiz.

Natijada quyidagi determinantni hosil qilamiz.

$$\Delta = \begin{vmatrix} 2 & -5 & 1 & 2 \\ -1 & 2 & 0 & 6 \\ 1 & 1 & 0 & 3 \\ 2 & -1 & 0 & 0 \end{vmatrix}$$

Hosil qilingan determinantni 3- ustun bo‘yicha yoyamiz.

$$\Delta = 1\cdot(-1)^{1+3}\cdot \begin{vmatrix} -1 & 2 & 6 \\ 1 & 1 & 3 \\ 2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 2 & 6 \\ 1 & 1 & 3 \\ 2 & -1 & 0 \end{vmatrix}$$

Bu determinantning 2-satr elementlarini 2-ga ko‘paytirib, 1- satr elementlaridan ayiramiz.

$$\Delta = \begin{vmatrix} -3 & 0 & 0 \\ 1 & 1 & 3 \\ 2 & -1 & 0 \end{vmatrix}$$

Bu determinantni 1-satr elementlari bo‘yicha yoyib natijani hosil qilamiz.

$$\Delta = -3\cdot(-1)^{1+1}\cdot \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = -3\cdot(1\cdot 0 - 3\cdot(-1)) = -9.$$

1.3. Teskari matritsa

Agar $\Delta_A = 0$ bo‘lsa A kvadrat matritsa xos matritsa, $\Delta_A \neq 0$ bo‘lsa, xosmas matritsa deyiladi.

Agar $A\cdot A^{-1} = A^{-1}\cdot A = E$ kabi bo‘lsa, A^{-1} kvadrat matritsa, o‘shanday tartibli *A* kvadrat matritsaga teskari matritsa deyiladi. Berilgan matritsaga teskari matritsa mavjud bo‘lishi uchun, berilgan matritsaning xosmas bo‘lishi zarur va etarlidir. Teskari matritsa quyidagi formuladan topiladi:

$$A^{-1} = \begin{pmatrix} \frac{A_{11}}{\Delta_A} & \frac{A_{21}}{\Delta_A} & \dots & \frac{A_{n1}}{\Delta_A} \\ \frac{A_{12}}{\Delta_A} & \frac{A_{22}}{\Delta_A} & \dots & \frac{A_{n2}}{\Delta_A} \\ \dots & \dots & \dots & \dots \\ \frac{A_{1n}}{\Delta_A} & \frac{A_{2n}}{\Delta_A} & \dots & \frac{A_{nn}}{\Delta_A} \end{pmatrix}.$$

10-misol. $A=\begin{pmatrix}1 & 3 & -5\\ 0 & 1 & 2\\ 0 & 0 & 1\end{pmatrix}$ matritsaga teskari matritsani toping.

Yechish. Birinchi ustun bo‘yicha yoyib A matritsaning determinantini hisoblaymiz.

$$\Delta_A = 1\cdot\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix} = 1 \neq 0$$

Demak, A matritsaga teskari matritsa mavjud.

A matritsaning algebraik to‘ldiruvchilarini topamiz:

$$A_{11}=\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}=1 \quad A_{21}=-\begin{vmatrix}3 & -5\\ 0 & 1\end{vmatrix}=-3 \quad A_{31}=\begin{vmatrix}3 & -5\\ 1 & 2\end{vmatrix}=11$$

$$A_{12}=-\begin{vmatrix}0 & 2\\ 0 & 1\end{vmatrix}=0 \quad A_{22}=\begin{vmatrix}1 & -5\\ 0 & 1\end{vmatrix}=1 \quad A_{32}=-\begin{vmatrix}1 & -5\\ 0 & 2\end{vmatrix}=-2$$

$$A_{13}=\begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix}=0 \quad A_{23}=-\begin{vmatrix}1 & 3\\ 0 & 0\end{vmatrix}=0 \quad A_{33}=\begin{vmatrix}1 & 3\\ 0 & 1\end{vmatrix}=1$$

Natijada:

$$A^{-1}=\frac{1}{1}\cdot\begin{pmatrix}1 & -3 & 11\\ 0 & 1 & -2\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & -3 & 11\\ 0 & 1 & -2\\ 0 & 0 & 1\end{pmatrix}.$$

1.4. Chiziqli algebraik tenglamalar tizimi

Chiziqli tenglama deb,

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b,$$

ko‘rinishdagi tenglamaga aytiladi, bu yerda a_i va b – sonlar, x_i - noma’lumlar. Shunday qilib, chiziqli tenglamaning chap tomonida no‘malumlarning chiziqli kombinatsiyasi, o‘ng tomonida esa son turadi.

Agar $b=0$ bo‘lsa, chiziqli tenglama **bir jinsli**, aks holda, ya’ni $b\neq 0$ bo‘lsa, **bir jinsli bo‘lmagan** tenglama deyiladi.

Chiziqli tenglamalar tizimi deb quyidagi ko‘rinishdagi tizimga aytiladi:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases}, \qquad (1)$$

bu yerda a_{ij}, b_i - sonlar, x_j - noma’lumlar, n – noma’lumlar soni, m – tenglamalar soni ($i = \overline{1,m};\ j = \overline{1,n}$).

Chiziqli tenglamalar tizimining yechimi deb Shunday $x_1, x_2, ..., x_n$, sonlarga aytiladiki, bu sonlarni noma’lumlar o‘rniga quyilganda, tizimning har bir tenglamasi o‘rinli tenglikka aylanadi.

Agar chiziqli tenglamalar tizimi hech bo‘lmaganda bitta yechimga ega bo‘lsa **birgalikda bo‘lgan**, aks holda, ya’ni yechimga ega bo‘lmasa, **birgalikda bo‘lmagan** tenglamalar tizimi deyiladi.

Shuningdek, agar birgalikda bo‘lgan tenglamalar tizimi yagona yechimga ega bo‘lsa **aniqlangan**, bittadan ko‘p echimga ega bo‘lsa, **aniqlanmagan** tenglamalar tizimi deb yuritiladi.

Gauss usuli

Faraz qilaylik (1) tizimda $a_{11} \neq 0$ bo‘lsin (bunga tenglamalar o‘rnini almashtirish yordamida doimo erishish mumkin). Birinchi tenglamani har ikkala tomonini avval a_{11} ga bo‘lamiz, so‘ngra a_{i1} ga ko‘paytiramiz, bu erda i – navbatdagi tenglama tartibi.

Hosil bo‘lgan tenglamalarni tizimning qolgan har bir mos tenglamasidan ayiramiz. Ma’lumki, hosil bo‘lgan yangi tenglamalar tizimi berilgan tenglamalar tizimiga teng kuchlidir. Hosil bo‘lgan yangi tizimda birinchi tenglamadan boshqa barcha tenglamalarda $x_1 = 0$ bo‘ladi, ya’ni tizim quyidagi ko‘rinishda bo‘ladi:

$$\begin{cases} x_1 + \tilde{a}_{12}x_2 + ... + \tilde{a}_{1n}x_n = \tilde{b}_1 \\ \quad \tilde{a}_{22}x_2 + ... + \tilde{a}_{2n}x_n = \tilde{b}_2 \\ \quad .... \\ \quad \tilde{a}_{n2}x_2 + ... + \tilde{a}_{nn}x_n = \tilde{b}_n \end{cases}.$$

Agar yangi hosil bo‘lgan tenglamalar tizimida x_2 ning oldidagi koffitsientlar hammasi nolga teng bo‘lmasa shu asnoda x_2 ni ham yo‘qotish mumkin.

Bu jarayonni tizimdagi barcha noma’lumlar uchun davom ettirib, berilgan tenglamalar tizimini quyidagi uchburchak shakliga keltirish mumkin:

$$\begin{cases} x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ \quad x_2 + ... + a_{2n}x_n = b_2 \\ \quad .... \\ \quad\quad x_n = b_n \end{cases} \qquad (2)$$

Bu erda $\tilde{a}_{ij}, a_{ij}, \tilde{b}_i$ va b_i belgilar bilan almashtirishlar natijasida hosil bo‘lgan sonli koeffitsientlar va ozod xadlar belgilangan. (2) tizimning oxirgi tenglamasidan x_n ni topamiz. So‘ngra ketma-ket o‘rniga qo‘yish yordamida qolgan noma’lumlarni topamiz.

11-misol.

Tenglamalar tizimini Gauss usulida eching:

$$\begin{cases} 3x - y + 2z = 9 \\ x + 4y + z = 4 \\ 2x - 3y + 3z = 11 \end{cases}.$$

Yechish. Gauss usuli berilgan tenglamalar tizimidagi noma’lumlarni ketma-ket yo‘qotishdan iboratdir. Bu usulni qo‘llash oson bo‘lishi uchun 1- va 2- tenglamalarning o‘rnini almashtiramiz.

$$\begin{cases} x+4y+z=4 \\ 3x-y+2z=9 \\ 2x-3y+3z=11 \end{cases}.$$

endi 2- va 3- tenglamalardan *x* ni yo‘qotamiz. Buning uchun birinchi tenglamani 3ga ko‘paytirib, ikkinchi tenglamadan, 2 ga ko‘paytirib, 3- tenglamadan ayiramiz va quyidagiga ega bo‘lamiz:

$$\begin{cases} x+4y+z=4 \\ -13y-z=-3 \\ -11y+z=3 \end{cases}.$$

2- tenglamaga 3- tenglamani qo‘shib, 3- tenglamadan *z* ni yo‘qotamiz:

$$\begin{cases} x+4y+z=4 \\ -13y-z=-3 \\ -24y=0 \end{cases}.$$

Oxirgi tenglamadan $y=0$ ekanligi kelib chiqadi. Bu qiymatni 2- tenglamaga qo‘yib *z* ni aniqlaymiz. Topilgan *y* va *z* ni 1-chi tenglamaga qo‘yib topamiz. z=3, x=1.

Shunday qilib, x = 1, y = 0, z = 3.

Kramer usuli

No‘malumlar soni tenglamalar soniga teng bo‘lgan quyidagi chiziqli tenglamalar tizimini qaraylik:

$$\begin{cases} a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\dots+a_{2n}x_n=b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1+a_{n2}x_2+\dots+a_{nn}x_n=b_n \end{cases} \qquad (3)$$

Elementlari noma’lumlar oldidagi koeffitsientlardan iborat Δ determinatni **asosiy determinant** deb ataymiz.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}, \qquad (4)$$

determinantda x_j noma'lumlar oldidagi koeffitsiyentlardan tuzilgan ustunni ozod hadlardan iborat ustun bilan almashtirishdan hosil bo'lgan determinantni Δ_{x_j} bilan belgilaymiz.

U holda: Agar $\Delta \neq 0$ bo'lsa, (3) tizim quyidagi formulalar bilan aniqlanuvchi yagona yechimga ega: $x_1 = \frac{\Delta_{x_1}}{\Delta}, x_2 = \frac{\Delta_{x_2}}{\Delta}, ..., x_n = \frac{\Delta_{x_n}}{\Delta}$.

1) Agar $\Delta = \Delta_{x_j}$ =0 bo'lsa, tizim cheksiz ko'p yechimga ega.

2) Agar Δ = 0, va Δ_{x_j} lardan hech bo'lmaganda bittasi noldan farqli bo'lsa, tizim yechimga ega emas.

12-misol.

Tenglamalar tizimini Kramer usulida yeching:

$$\begin{cases} 4x - y + z = 2 \\ x + y - 2z = 1 \\ 2x + 3y - 4z = 6 \end{cases}$$

Yechish. Asosiy determinantni hisoblaymiz:

$$\Delta = \begin{vmatrix} 4 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3 & -4 \end{vmatrix} = 9 \neq 0,$$

demak, tenglamalar tizimi yagona yechimga ega.

Δ_x, Δ_u va Δ_z– larni topamiz.

$$\Delta_x = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \\ 6 & 3 & -4 \end{vmatrix} = 9, \quad \Delta_y = \begin{vmatrix} 4 & 2 & 1 \\ 1 & 1 & -2 \\ 2 & 6 & -4 \end{vmatrix} = 36, \quad \Delta_z = \begin{vmatrix} 4 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 6 \end{vmatrix} = 18.$$

Bundan

$$x = \frac{\Delta_x}{\Delta} = \frac{9}{9} = 1, \quad y = \frac{\Delta_y}{\Delta} = \frac{36}{9} = 4, \quad \Delta_z = \frac{\Delta_z}{\Delta} = \frac{18}{9} = 2.$$

Chiziqli tenglamalar tizimini teskari matritsa yordamida yechish

Chiziqli tenglamalar tizimi (3)-ni qaraylik va quyidagicha belgilashlar kiritaylik:

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$ - tizimning matritsasi,

$$X=\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}$$ - noma’lumlar ustuni,

$$B=\begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}$$ - ozod hadlar ustuni. U holda (3) tizimni matritsaviy tenglama

ko‘rinishida quyidagicha yozish mumkin:

$$AX = B. \qquad (5)$$

Faraz qilaylik A - xosmas matritsa bo‘lsin, u holda unga teskari A^{-1} matritsa mavjud bo‘ladi. (5) tenglamaning har ikki tomonini A^{-1} ga chapdan ko‘paytiraylik.

$$A^{-1}AX = A^{-1}B.$$

Ma’lumki $A^{-1}A=E,$ u holda $EX = A^{-1}B$, $EX = X$ ekanligidan $X = A^{-1}B.$

Shunday qilib, (5) – matritsaviy tenglamaning yechimi, A matritsaga teskari matritsaning (3) tizimning ozod hadlaridan iborat ustun matritsaga ko‘paytmasiga teng ekan.

13-misol.

Tenglamalar tizimini teskari matritsa yordamida yeching.

$$\begin{cases} x-3y+z=1 \\ 2x+y-z=6 \\ 5x-4y-7z=4 \end{cases}$$

Yechish. Tizimning matritsasini tuzamiz.

$$A=\begin{pmatrix} 1 & -3 & 1 \\ 2 & 1 & -1 \\ 5 & -4 & -7 \end{pmatrix}$$

Δ_A = -51 ≠ 0, demak, tenglamalar tizimi yagona yechimga ega.

A^{-1} matritsani topamiz:

$$A_{11}=-11 \quad A_{21}=-25 \quad A_{31}=2$$

$$A_{12}=9 \quad A_{22}=-12 \quad A_{32}=3$$

$$A_{13}=-13 \quad A_{23}=-11 \quad A_{33}=7$$

U xolda $A^{-1}=-\frac{1}{51}\begin{pmatrix} -11 & -25 & 2 \\ 9 & -12 & 3 \\ -13 & -11 & 7 \end{pmatrix}$.

Agar $B=\begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}, \quad X=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ‘ekanligini e’tiborga olsak, berilgan tenglamalar tizimi yechimi $X = A^{-1}B$ bo‘lgan $AX = B$ matritsaviy tenglamaga aylanadi.

Shunday qilib,

$$X=-\frac{1}{51}\begin{pmatrix} -11 & -25 & 2 \\ 9 & -12 & 3 \\ -13 & -11 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}=-\frac{1}{51}\begin{pmatrix} -11-150+8 \\ 9-72+12 \\ -13-66+28 \end{pmatrix}=-\frac{1}{51}\begin{pmatrix} -153 \\ -51 \\ -51 \end{pmatrix}=\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix},$$

ya’ni $x = 3$, $y = 1$, $z = 1$.

Topshiriq variantlari

Hisob topshiriqlarini qabul qilishda beriladigan nazariy savollar:

1. Ikkinchi va uchinchi tartibli determinantlar.
2. Determinantlarning xossalari.
3. Minorlar va algebraik to‘ldiruvchilar
4. Yuqori tartibli determinantlar.
5. Chiziqli tnglamalar sistemasini yechish uchun Kramer usuli.
6. Matritsa tushunchasi.
7. Matritsalar ustida amallar. Matritsalarni qo‘shish va ayirish.
8. Matritsalarni ko‘paytirish
9. Teskari matritsa.
10. Chiziqli tnglamalar sistemasini teskari matritsa usulida yechish.

Chiziqli algebra bo‘limidan topshiriq variantlari

1-variant

1. Determinantni hisoblang: $\begin{vmatrix} 8 & 1 & 9 & 0 \\ 6 & -1 & 4 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & -1 & 2 & -2 \end{vmatrix}$.

2. $A=\begin{pmatrix} 3 & 0 & 4 \\ -2 & 2 & -3 \\ 1 & 1 & 2 \end{pmatrix}$ va $B=\begin{pmatrix} -1 & 1 & 2 \\ 0 & 1 & -2 \\ 5 & 3 & 1 \end{pmatrix}$ matritsalar uchun $A^2-BA+3A$ matritsali ko'phadni hisoblang.

3. $\begin{pmatrix} 1 & -1 & 3 \\ 3 & -5 & 1 \\ 4 & -7 & 1 \end{pmatrix}$ matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x-y+5t=6 \\ 3x+2y-z=3 \\ -x+2y+4z+t=10 \\ -y-z+3t=0 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x+2y-2z=5 \\ 4x-y+10z=11. \\ 5x+3y-5z=9 \end{cases}$$

2-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{vmatrix}$.

2. $A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & -1 & 2 \\ 1 & -1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -7 & 1 & -3 \\ 5 & 1 & 2 \\ 0 & 1 & 4 \end{pmatrix}$ matritsalar uchun $B^2 + BA + 2A$ matritsali ko‘p hadni hisoblang

3. $\begin{pmatrix} 8 & 5 & -46 \\ 2 & 1 & -12 \\ 3 & 2 & 25 \end{pmatrix}$ matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - 3z + 4t = -4 \\ 2x + y + 10z - 15t = 10 \\ 2y + 3z - 6t = 7 \\ 3x + 4y - z + 2t = 4 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x_1 - x_2 - x_3 - 2x_4 = -1 \\ -x_1 - 2x_2 + 3x_3 + x_4 = 3 \\ x_1 + x_2 - 2x_3 - x_4 = 2 \\ 2x_1 - 3x_2 + x_3 - 2x_4 = 1 \end{cases}$$

3-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 1 & 4 & 1 \\ 2 & 1 & 3 & 0 \\ 3 & 1 & 2 & 1 \\ 4 & 1 & 1 & 0 \end{vmatrix}.$

2. $A = \begin{pmatrix} 4 & 1 & 2 \\ -2 & 0 & 2 \\ 3 & -1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -2 & 4 \\ 1 & -2 & -4 \end{pmatrix}$ matritsalaruchun $A^2 - 2BA + A$ matritsaviy ko‘phadni hisoblang

3. $\begin{pmatrix} 3 & 1 & 6 \\ 2 & -3 & 6 \\ 5 & 1 & 27 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x - y + 5t = 6 \\ 3x + 2y - z = 3 \\ -x + 2y + 4z + t = 10 \\ -y - z + 3t = 0 \end{cases}.$$

5. Tenglamalar tizimini matritsa usulida yeching:

$$\begin{cases} x - y = 3 \\ 2x + y - 3z = 3 \\ -x - 2y + 3z = 0 \end{cases}.$$

4-variant

1. Determinantni hisoblang: $\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}.$

2. $A = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 1 & 2 \\ -3 & 3 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 2 & 4 \\ 3 & 1 & -1 \end{pmatrix}$ matritsalar uchun $2A^2 + BA + 3A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} -5 & 3 & 14 \\ 4 & 2 & 13 \\ 3 & 5 & 26 \end{pmatrix}.$ matritsaga teskari matritsani hisoblang.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 4x + 4y - 5z = -2 \\ 3x + 2y + z = 7 \\ x - y + 10z = 20 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x + 3y - z = 4 \\ x + 2y + z = 1 \\ x + 4y - 3z = 7 \end{cases}.$$

5-variant

1. Determinantni hisoblang: $\begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & 1 \\ 0 & 2 & 1 & -3 \\ 6 & -2 & 9 & 8 \end{vmatrix}$.

2. $A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & -3 \\ 5 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & -2 \\ 5 & -4 & 1 \end{pmatrix}$ matritsalar uchun $B^2 - BA + 4A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 3 & 4 & 27 \\ 4 & -1 & 35 \\ 5 & -2 & 43 \end{pmatrix}$. matritsaga teskari matritsani hisoblang.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x + 3y - 4z + 5t = 3 \\ -y - t = -1 \\ x - 3z + 8t = -1 \\ x + 2y - 4z + 3t = 0 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 2x_1 - 3x_2 + x_3 = -3 \\ x_1 + 2x_2 - 3x_3 = -5 \\ 5x_1 + x_2 - 6x_3 = -16 \end{cases}$$

6-variant

1. 1. Determinantni hisoblang: $\begin{vmatrix} 5 & 6 & 0 & 0 & 0 \\ 1 & 5 & 6 & 0 & 0 \\ 0 & 1 & 5 & 6 & 0 \\ 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 & 5 \end{vmatrix}$.

2. . $A = \begin{pmatrix} 4 & 0 & 2 \\ -1 & 1 & -3 \\ 5 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 1 & -2 \\ 5 & -5 & 0 \end{pmatrix}$ matritsalar uchun $A^2 + BA + 3B$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 2 & -1 & -3 \\ 3 & 2 & -4 \\ 2 & -3 & 5 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x_1 - x_2 + 2x_3 - 3x_4 = 4 \\ -x_1 + 2x_2 - x_3 + 4x_4 = 1 \\ 2x_1 + x_2 - 2x_3 + 4x_4 = 1 \\ x_1 + x_2 + x_3 + x_4 = 7 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - 3y + 2z = 2 \\ x + y - 5z = 7 \\ 3x - y - 8z = 16 \end{cases}.$$

7-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \\ 4 & 1 & 1 & 0 \end{vmatrix}$.

2. $A = \begin{pmatrix} 1 & 5 & 4 \\ -2 & 2 & -4 \\ 1 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -5 & 1 & 2 \\ 0 & 3 & -1 \\ 2 & 3 & 1 \end{pmatrix}$ matritsalar uchun

$A^2 - BA + 4B$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 2 & 4 & 3 \\ 3 & 12 & 5 \\ 4 & 1 & -1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 7x - 2y + 4z = 13 \\ 2x + 2y - z = 2 \\ 3x - y + z = 0 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x_1 - x_2 - x_3 - 2x_4 = -1 \\ -x_1 - 2x_2 + 3x_3 + x_4 = 8 \\ x_1 + x_2 - 2x_3 - x_4 = -5 \\ 2x_1 - 3x_2 + x_3 - 2x_4 = 5 \end{cases}$$

8-variant

1. Determinantni hisoblang : $\begin{vmatrix} 2 & 0 & -1 & 1 & 0 \\ 1 & 2 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 0 & 2 \end{vmatrix}$.

2. $A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 2 & 3 \\ 3 & -7 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 5 & 2 \\ 5 & 3 & 1 \end{pmatrix}$ matritsalar uchun

$B^2 - BA + 3A$. matritsali ko‘phadni hisoblang

3. $\begin{pmatrix} 2 & 3 & 1 \\ 0 & 6 & 6 \\ -1 & -2 & -1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x - y + 5t = 6 \\ 3x + 2y - z = 3 \\ -x + 2y + 4z + t = 10 \\ -y - z + 3t = 0 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 2x + y - 3z = 5 \\ x - y + 2z - 2t = -4. \\ 2y - z - t = 3 \end{cases}$$

9-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 2 & 1 & 4 \\ 8 & 0 & 1 & 9 \\ -9 & 1 & 1 & -7 \\ 3 & -1 & 1 & 3 \end{vmatrix}$.

2. $A = \begin{pmatrix} 3 & -1 & 4 \\ 3 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 1 & 2 \\ -4 & 0 & 2 \\ 2 & -4 & 3 \end{pmatrix}$ matritsalar uchun $A^2 + 3BA + 2B$ matritsali ko‘phadni hisoblang.

3. . $\begin{pmatrix} 3 & 5 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x + 3y - 4z + 5t = 3 \\ -y - t = -1 \\ x - 3z + 8t = -1 \\ x + 2y - 4z + 3t = 0 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - 2y - z = 2 \\ -2x - 4y + 2z = 4 \\ 2x + y - 3z = -2 \end{cases}$$

10-variant

1. Determinantni hisoblang: $\begin{vmatrix} 3 & 3 & -4 & -3 \\ 0 & 6 & 1 & 1 \\ 5 & 4 & 2 & 1 \\ 2 & 3 & 3 & 2 \end{vmatrix}$.

2. $A = \begin{pmatrix} -1 & 0 & 4 \\ 2 & -3 & 1 \\ 1 & 1 & -5 \end{pmatrix}$ va $B = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 6 & -2 \\ 2 & 3 & 0 \end{pmatrix}$ matritsalar uchun $A^2 - BA + 3A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 3 & 2 & 5 \\ -5 & 4 & 3 \\ 1 & -3 & -1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x + y = 1 \\ y + z = 4. \\ x + z = 6 \end{cases}$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x + y - 5z - t = 2 \\ x - 2y + 2t = 1 \\ -x + 3y - z - 3t = -1 \\ x - y - z + t = 1 \end{cases}.$$

11-variant

1. Determinantni hisoblang : $\begin{vmatrix} 3 & 1 & 1 & 4 \\ 0 & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{vmatrix}$.

2. $A = \begin{pmatrix} 3 & -5 & 4 \\ 2 & 0 & 3 \\ 1 & 1 & -4 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 2 \\ 1 & 3 & -3 \end{pmatrix}$ matritsalar uchun $B^2 - BA + 2A$ matritsali ko‘phadni hisoblang

3. $\begin{pmatrix} 2 & 4 & 3 \\ 3 & -1 & 4 \\ 4 & -2 & 5 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x - z = -2 \\ 2x - y - z = 4 \\ y - z = -6 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 2x + y - 5z - t = 2 \\ x - 2y + 2t = 1 \\ -x + 3y - z - 3t = -1 \\ x - y - z + t = 1 \end{cases}.$$

12-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 2 & -1 & 0 \\ 3 & -2 & 1 & 2 \\ 7 & 1 & 4 & 1 \\ 0 & 1 & 1 & -1 \end{vmatrix}$.

2. $A = \begin{pmatrix} 1 & 2 & 2 \\ -1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 1 & -3 \\ 5 & 0 & 2 \\ -5 & 3 & 1 \end{pmatrix}$ matritsalar uchun $3A^2$ – BA + +B matritsali ko‘phadni hisoblang

3. $\begin{pmatrix} 3 & 3 & 1 \\ 2 & -4 & -3 \\ 5 & 7 & 1 \end{pmatrix}$.matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - 2y + z + 3t = -6 \\ -10z + 2t = -2 \\ 2x + 2y - 5z - 2t = 8 \\ x + y - z = 3 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching :

$$\begin{cases} 4x - y + z = 1 \\ -3x + 2y + 5z = -20. \\ -4x - 2y + z = -18 \end{cases}$$

13-variant

1. Determinantni hisoblang: $\begin{vmatrix} 7 & 1 & -1 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & -1 & 1 & -1 \\ 2 & 0 & 1 & 5 \end{vmatrix}$.

2. $A = \begin{pmatrix} 3 & 3 & 4 \\ -2 & -4 & -3 \\ 1 & 3 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 5 & 3 & 4 \end{pmatrix}$ matritsalar uchun $A^2 + 4BA + B$ matritsali ko‘phadni hisoblang

3. $\begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & 3 \\ 1 & -3 & 1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x - y + z = 0 \\ 2x + y - 2z = -1. \\ 2x - y + z = 0 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 5x + y - 4z = 2 \\ x + 2y - z = 1 \\ 3x - 3y - 2z = 0 \end{cases}.$$

14-variant

1. Determinantni hisoblang: $\begin{vmatrix} 2 & 1 & 0 & -1 \\ 3 & 7 & 1 & 1 \\ 2 & 2 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{vmatrix}$.

2. $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & -3 \\ -1 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & 1 \\ 5 & -3 & 1 \end{pmatrix}$ matritsalar uchun $A^2 - BA + 2B$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 3 & -2 & -7 \\ 3 & 4 & -1 \\ 2 & -1 & -1 \end{pmatrix}$. matritsaga teskari matritsani toping

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - y - z = 1 \\ 2x + 2y - z = 2. \\ -y + 4z = 0 \end{cases}$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x + y - 5z - t = 2 \\ x - 2y + 2t = 1 \\ -x + 3y - z - 3t = -1 \\ z - y - z + t = 1 \end{cases}$$

15-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 2 & 1 & -1 \\ -1 & 0 & 2 & -2 \\ 1 & 3 & 2 & 3 \\ 1 & -1 & 5 & -1 \end{vmatrix}$.

2. $A = \begin{pmatrix} 2 & 0 & 4 \\ 3 & -1 & 3 \\ 4 & 1 & -5 \end{pmatrix}$ va $B = \begin{pmatrix} 2 & 1 & 2 \\ -1 & 0 & 4 \\ 1 & -3 & 1 \end{pmatrix}$ matritsalar uchun $B^2 - BA + 5A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 2 & 5 & -3 \\ 3 & 13 & -5 \\ 2 & 7 & 4 \end{pmatrix}$. matritsaga teskari matritsani toping

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + y + z = 1 \\ 2x - 2y + 3z = 3. \\ 4x - y = 0 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x + 2y - z - 2t = 5 \\ -2x - y + 2z + t = -4 \\ -x + y + z - t = 1 \\ x - y - z + t = -1 \end{cases}.$$

16-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 4 & -1 \\ 1 & 2 & 2 & 1 \\ 7 & 0 & 5 & 1 \end{vmatrix}$.

2. $A = \begin{pmatrix} 0 & 3 & -4 \\ -1 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 7 \\ -5 & 3 & 1 \end{pmatrix}$ matritsalar uchun $A^2 - BA + 3A$

matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 3 & -2 & -5 \\ 2 & 17 & 4 \\ 5 & 16 & 3 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 12x_1 + 13x_2 - 10x_3 - 11x_4 = 6 \\ 10x_1 - 5x_2 + 7x_3 - 3x_4 = 1 \\ 11x_1 - 5x_2 + 10x_3 - 5x_4 = 1 \\ 7x_1 + x_2 - 6x_3 + 2x_4 = 7 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x + y - 5z - t = 2 \\ x - 2y + 2t = 1 \\ -x + 3y - z - 3t = -1 \\ x - y - z + t = 1 \end{cases}.$$

17-variant

1. Determinantni hisoblang: $\begin{vmatrix} 3 & 2 & 5 & 1 \\ -5 & 4 & 3 & 0 \\ 1 & -3 & -1 & -1 \\ 3 & -2 & 4 & 0 \end{vmatrix}.$

2. $A = \begin{pmatrix} 3 & 0 & 3 \\ 1 & -2 & 3 \\ 1 & 2 & -1 \end{pmatrix}$ va $B = \begin{pmatrix} 2 & 1 & 2 \\ 0 & -2 & 3 \\ -1 & 3 & -2 \end{pmatrix}$ matritsalar uchun

A^2 – BA + 2B matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 28 & 3 & 4 \\ 7 & 4 & -1 \\ 14 & 5 & -2 \end{pmatrix}.$ matritsaga teskari matritsani toping

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x - 2y + 5z = 20 \\ 3x + 4y + 4z = -13. \\ x + 2y + z = -8 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x_1 + x_2 + 3x_3 - 2x_4 = 2 \\ 2x_1 + 2x_2 + 4x_3 - x_4 = 5 \\ 3x_1 + 3x_2 + 5x_3 - 2x_4 = 6 \\ 2x_1 + 2x_2 + 8x_3 - 3x_4 = 7 \end{cases}$$

18-variant

1. Determinantni hisoblang : $\begin{vmatrix} -5 & 3 & 14 & 0 \\ 4 & 2 & 13 & -1 \\ 3 & 5 & 26 & 1 \\ 0 & 0 & 1 & -2 \end{vmatrix}.$

2. $A=\begin{pmatrix} 3 & 0 & 1 \\ 1 & -4 & 3 \\ 0 & 2 & 2 \end{pmatrix}$ va $B=\begin{pmatrix} 2 & 1 & 0 \\ -3 & -1 & -4 \\ 2 & 0 & 1 \end{pmatrix}$ matritsalar uchun

$A^2 - 2BA + 3B$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 2 & -1 & 3 \\ 3 & -5 & 1 \\ 4 & -7 & 1 \end{pmatrix}.$ matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 3x-2y+z=2 \\ 4x+y+5z=10 \\ -x+10y-z=8 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x+y-5z-t=2 \\ x-2y+2t=1 \\ -x+3y-z-3t=-1 \\ x-y-z+t=1 \end{cases}.$$

19-variant

1. Determinantni hisoblang : $\begin{vmatrix} 2 & 3 & -4 & 5 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & -3 & 8 \\ 1 & 2 & -4 & 3 \end{vmatrix}$.

2. $A = \begin{pmatrix} 3 & 1 & 3 \\ -1 & 2 & 0 \\ 1 & -1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 4 & -1 & 2 \\ 0 & 1 & 2 \\ 5 & -2 & 1 \end{pmatrix}$ matritsalar

chun $B^2 - BA + 3A$matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 2 & -1 & 0 & 5 \\ 3 & 2 & -1 & 0 \\ -1 & 2 & 4 & 1 \\ 0 & -1 & 1 & 3 \end{pmatrix}$. matritsaga teskari matritsani toping

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + y + z = 1 \\ 2x - 2y + 3z = 3. \\ 4x - y = 0 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x_1 + 2x_2 - x_3 - 2x_4 = 5 \\ -2x_1 - x_2 + 2x_3 + x_4 = -4 \\ -x_1 + x_2 + x_3 - x_4 = 1 \\ x_1 - x_2 - x_3 + x_4 = -1 \end{cases}.$$

20-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & -1 & 2 & -3 \\ -1 & 2 & -1 & 4 \\ 2 & 1 & -2 & 5 \\ 1 & 1 & 1 & 1 \end{vmatrix}$.

2. $A = \begin{pmatrix} 3 & 0 & -2 \\ 3 & 1 & 1 \\ -3 & -1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 3 & 0 & 1 \end{pmatrix}$ matritsalar uchun $B^2 - BA + 4B$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 1 & 2 & -2 \\ 4 & -1 & 10 \\ 5 & 3 & -5 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - y + z = 1 \\ x + 2y - z = 2. \\ 2x + z = 0 \end{cases}$$

5. Tenglamalar tizimini Gauss usulida yeching

$$\begin{cases} 2x_1 + x_2 - 5x_3 - x_4 = 2 \\ x_1 - 2x_2 + 2x_4 = 1 \\ -x_1 + 3x_2 - x_3 - 3x_4 = -1 \\ x_1 - x_2 - x_3 + x_4 = 1 \end{cases}.$$

21-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 0 & -3 & 4 \\ 2 & 1 & 10 & -15 \\ 0 & 2 & 3 & -6 \\ 3 & 4 & -1 & 2 \end{vmatrix}$.

2. $A = \begin{pmatrix} 0 & 1 & 3 \\ 1 & -2 & -1 \\ 1 & 3 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 2 & -1 & 0 \\ 2 & 1 & -3 \\ 5 & -2 & 1 \end{pmatrix}$ matritsalar uchun $B^2 + BA + 3B$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 1 & 2 & -2 \\ 4 & -1 & 10 \\ 5 & 3 & -5 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 8x - 7y + 10z - 18t = 17 \\ 3x + 4y + 9z - 10t = 7 \\ 2x - 5y + 7z - 10t = 11 \\ 9x + 8y + 4z - 7t = 2 \end{cases}.$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 5x_1 + x_2 - 8x_3 + 10x_4 = -5 \\ 2x_1 + x_2 - 2x_3 + x_4 = 1 \\ -3x_1 - 2x_2 + 2x_3 + x_4 = -4 \\ x_1 + 2x_2 + 2x_3 - 7x_4 = 8 \end{cases}.$$

22-variant

1. Determinantni hisoblang: $\begin{vmatrix} 3 & 4 & 1 & 2 \\ 5 & 7 & 1 & 3 \\ 4 & 5 & 2 & 1 \\ 7 & 10 & 1 & 6 \end{vmatrix}$.

2. $A = \begin{pmatrix} -2 & 1 & 3 \\ 1 & 2 & 0 \\ -1 & -3 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & -1 & 2 \\ 0 & 4 & -3 \\ 3 & -2 & 1 \end{pmatrix}$ matritsalar uchun $\mathrm{B}^2 - 2\mathrm{B}A + 4A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 3x - y + 2z = 4 \\ 2x + 3y - 4z = 3. \\ -x + 2y - 2z = 1 \end{cases}$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 12x_1 + 14x_2 - 15x_3 + 24x_4 = 22 \\ 16x_1 + 18x_2 - 22x_3 + 29x_4 = 27 \\ 18x_1 + 20x_2 - 21x_3 + 32x_4 = 30 \\ 10x_1 + 12x_2 - 16x_3 + 20x_4 = 18 \end{cases}$$

23-variant

1. Determinantnti hisoblang: $\begin{vmatrix} 1 & 2 & -1 & 2 \\ -2 & -1 & 2 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \end{vmatrix}$.

2. $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 0 \\ 2 & 3 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 5 & 1 & -3 \\ 0 & -1 & 2 \\ 4 & 2 & 1 \end{pmatrix}$ matritsalar uchun $B^2 + 2BA + A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -2 \\ 2 & -1 & 1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + 2y - 4z = -9 \\ -x - 3y + 6z = 13. \\ 2x + 5y - z = -4 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} -3x_1 + x_2 + x_3 = 6 \\ 2x_1 + 7x_2 + 30x_3 = 65 \\ -5x_1 - 2x_2 - 13x_3 = 5 \end{cases}$$

24-variant

1. Determinantni hisoblang: $\begin{vmatrix} 12 & 13 & -10 & -11 \\ 10 & -5 & 7 & -3 \\ 11 & -5 & 10 & -5 \\ 7 & 1 & -6 & 2 \end{vmatrix}$.

2. $A = \begin{pmatrix} 0 & 1 & 4 \\ 5 & -1 & 0 \\ 2 & 3 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 1 & -3 \\ 0 & 2 & 4 \\ -3 & 2 & 1 \end{pmatrix}$ matritsalar uchun $B^2 - 5BA + 2A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 1 & 2 & -4 \\ -1 & -3 & 6 \\ 2 & 5 & -1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} x - 3z + 4t = -4 \\ 2x + y + 10z - 15t = 10 \\ 2y + 3z - 6t = 7 \\ 3x + 4y - z + 2t = 4 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + 2y - z - 2t = 5 \\ -2x - y + 2z + t = -4 \\ -x + y + z - t = 1 \\ x - y - z + t = -1 \end{cases}.$$

25-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 4 & 2 & 0 \\ 3 & 3 & 1 & 0 \\ -1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 2 \end{vmatrix}$.

2. $A = \begin{pmatrix} 1 & -1 & 4 \\ 0 & 2 & 0 \\ -3 & -2 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 1 & -3 \\ 3 & 4 & 5 \\ -2 & 2 & -1 \end{pmatrix}$ matritsalar uchun $B^2 + BA + 4A$ matritsali ko‘phadni hisoblang.

3. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 5 \\ 2 & 5 & 7 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x - 2y + 3z = -2 \\ -4x + 5y + 6z = -10. \\ x - y + z = 0 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 2x_1 - x_2 - x_3 - 2x_4 = 1 \\ -x_1 - 2x_2 + 3x_3 + x_4 = 2 \\ x_1 + x_2 - 2x_3 - x_4 = -1 \\ 2x_1 - 3x_2 + x_3 - 2x_4 = 3 \end{cases}.$$

26-variant

1. Determinantni hisoblang: $\begin{vmatrix} 2 & 2 & 0 & 1 \\ 2 & 1 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 5 & 2 & 1 & 0 \end{vmatrix}.$

2. Agar $X\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{pmatrix}.$ bo‘lsa, X matritsani toping.

3. $\begin{pmatrix} 4 & -1 & 2 \\ 1 & 1 & -2 \\ 0 & -1 & 3 \end{pmatrix}.$ matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 2x - y - z = 4 \\ 3x + 4y - 2z = 11. \\ 3x - 2y + 4z = 11 \end{cases}$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + 2y - 4z = 1 \\ 2x + y - 5z = -1. \\ x - y - z = -2 \end{cases}.$$

27-variant

1. Determinantni hisoblang: $\begin{vmatrix} 2 & 0 & 3 & 1 \\ -1 & -3 & 1 & 0 \\ 3 & 0 & 4 & 1 \\ 3 & 2 & 2 & 2 \end{vmatrix}$.

2. Agar $\begin{pmatrix} -3 & 1 & 2 \\ 1 & 0 & -1 \\ -4 & 3 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} -2 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & -1 & 4 \end{pmatrix}$. bo‘lsa, X matritsani toping.

3. $\begin{pmatrix} 1 & 5 & 1 \\ 3 & 2 & 1 \\ 6 & -2 & 1 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + y + 2z = -1 \\ 2x - y + 2z = -4. \\ 4x + y + 4z = -2 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 3x + 2y - z = 4 \\ 6x + 2y - 3z = 5. \\ 9x + 4y - 4z = 9 \end{cases}.$$

28-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 0 & 3 \\ 6 & 3 & 1 & -3 \\ 3 & 3 & 1 & -2 \end{vmatrix}$.

2. Agar $\begin{pmatrix} 2 & 1 & 0 \\ -15 & -3 & -1 \\ 2 & -3 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ -10 & -2 & -1 \end{pmatrix} \cdot$ bo‘lsa, X matritsani toping.

3. $\begin{pmatrix} 4 & 3 & 5 \\ 3 & 1 & 1 \\ 4 & 4 & 7 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching

$$: \begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases} .$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 2x + 3y + 2z = 8 \\ 5x - 8y + 2z = 17 \\ 7x - 5y + 4z = 25 \end{cases} .$$

29-variant

1. Determinantni hisoblang: $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 4 & 6 & 5 & 1 \end{vmatrix}$.

2. Agar $X \cdot \begin{pmatrix} 3 & -2 & 4 \\ 7 & 2 & 3 \\ 10 & -1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$. bo‘lsa, X matritsani toping.

3. $\begin{pmatrix} 5 & 2 & 5 \\ 3 & 5 & -3 \\ -2 & -4 & 3 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} x + 2y + 4z = 31 \\ 5x + y + 2z = 29. \\ 3x - y + z = 10 \end{cases}$$

5. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 2x + 5y - 3z = 15 \\ x + 2y + 2z = 7 \\ x + 3y - 5z = 8 \end{cases}.$$

30-variant

1. Determinantni hisoblang: $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$.

2. Agar $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$. bo‘lsa, X matritsani toping.

3. $\begin{pmatrix} 3 & 1 & 3 \\ 5 & -2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$. matritsaga teskari matritsani toping.

4. Tenglamalar tizimini matritsalar usulida yeching:

$$\begin{cases} 3x - y + z = 8 \\ x + 4y - 2z = 0 \\ 5x + 5y - 3z = 9 \end{cases}.$$

5. Tenglamalar tizimini Gauss usulida yeching:

$$\begin{cases} 4x + 2y + 3z = 10 \\ 2x + 3y - 2z = 7 \\ 2x - y + 5z = 3 \end{cases}.$$

II. Vektorlar algebrasi

2.1. Vektor tushunchasi. Vektorning uzunligi. O‘zlarining son qiymati va yo‘nalishi bilan aniqlanadigan miqdorlar vektorlar deb ataladi. $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtalar mos ravishda

$\bar{a}$ vektorning boshi va oxiri bo‘lsin. U holda $\bar{a}$ vektorning koordinatalari quyidagicha aniqlanadi.

$$\bar{a}=\overline{M_1M_2}=(x_2-x_1,y_2-y_1,z_2-z_1)$$

$\bar{a}$ vektorning uzunligiga teng bo‘lgan son uning moduli deyiladi va quyidagicha aniqlanadi.

$$|\bar{a}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Agar $\bar{a}$ vektor koordinata o‘qlari bilan mos ravishda α,β va γ burchaklar hosil qilsa, u holda $\cos\alpha,\cos\beta$ va $\cos\gamma$, $\bar{a}$ vektorning yo‘naltiruvchi kosinuslari deyiladi va quyidagicha aniqlanadi:

$$\cos\alpha=\frac{X}{|\bar{\alpha}|};\ \cos\beta=\frac{Y}{|\bar{\alpha}|};\ \cos\gamma=\frac{Z}{|\bar{\alpha}|}$$

Bu yerda: $X=x_2-x_1, Y=y_2-y_1, Z=z_2-z_1$

Vektorning o‘qqa proyeksiyasi

$\bar{a}$ vektorning U o‘qqa proyeksiyasi, uning moduli va U o‘q bilan tashkil qilgan burchagi φ orqali quyidagicha aniqlanadi. $np_U\,\bar{a}=|\bar{a}|\cos\varphi$

Ixtiyoriy $\bar{a}$ vektorning berilgan koordinatalar sistemasiga proyeksiyasini X,Y,Z orqali belgilaylik. U holda $\bar{a}=X,Y,Z$ va $|\bar{a}|=\sqrt{X^2+Y^2+Z^2}$ bo‘ladi.

1-misol. $\bar{a}$ =(6;3;-2) vektorning modulini toping.

Yechish. Modulni topish formulasiga asosan

$$\bar{a} = \sqrt{6^2 + 3^2 + (-2)^2} = \sqrt{36+9+4} = \sqrt{49} = 7$$

2-misol. A(3;-1;2) vaB(-1;2;1) nuqtalar berilgan. $\overline{AB}$ vektorning koordinatalarini toping.

Yechish: $\overline{AB}$ vektorning koordinatalarini topish uchun mos ravishda *B* nuqtaning koordinatalaridan *A* nuqtaning koordinatalarini ayiramiz.

$$\overline{AB} = (-1-3; 2-(-1); 1-2) = (-4;3;-1)$$

3-misol. $\bar{a} = (12;-15;-16)$ vektorning yo‘naltiruvchi kosinuslarini aniqlang.

Yechish. $\bar{a} = \sqrt{(12^2 + (-15)^2 + (-16)^2} = \sqrt{144+225+256} = 25$

Endi x=12; y=-15; z=-16 ekanligini e’tiborga olib yo‘naltiruvchi kosinuslarni aniqlaymiz.

$$\cos\alpha = \frac{12}{25};\ \cos\beta = -\frac{15}{25} = -\frac{3}{5};\ \cos\gamma = -\frac{16}{25}$$

2.2. Vektorlar ustida amallar

Vektorlarni qo‘shish va ayirish: Agar $\bar{a}$ va $\bar{b}$ vektorlar koordinatalari berilgan bo‘lsa, ya’ni $\bar{a} = (x_1, y_1, z_1)$ va $\bar{b} = (x_2, y_2, z_2)$ u holda

$$\bar{a} + \bar{b} = (x_1 + x_2; y_1 + y_2; z_1 + z_2)$$

$$\bar{a} - \bar{b} = (x_1 - x_2; y_1 - y_2; z_1 - z_2)$$

Vektorlarni songa ko‘paytirish.

Agar $\bar{a} = (x_1, y_1, z_1)$ bo‘lsa, u holda har qanday a son uchun quyidagi formula o‘rinli $\alpha\bar{a} = (\alpha x_1, \alpha y_1, \alpha z_1)$.

Vektorlarning kolleniarlik sharti.

Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotuvchi vektorlar kolleniar vektorlar deb ataladi.

$\bar{a} = (x_1, y_1, z_1)$ va $\bar{b} = (x_2, y_2, z_2)$ vektorlarning kolleniarlik sharti quyidagicha bo‘ladi:

$$\frac{x_2}{x_1}=\frac{y_2}{y_1}=\frac{z_2}{z_1}$$

Vektorlarni bazis koordinatalari bo‘yicha yoyish

$\bar{i}$, $\bar{j}$, $\bar{k}$ uchlik vektorlar bazis koordinatalari deyiladi, agar quyidagi uchta shart bajarilsa,

1) $\bar{i}$ vektor OX o‘qida, $\bar{j}$ vektor OY o‘qida, $\bar{k}$ vektor OZ o‘qida yotadi.

2)har bir $\bar{i}$, $\bar{j}$, $\bar{k}$ vektorlar o‘z o‘qlarida musbat tomonga yo‘nalgan bo‘ladi.

3) $\bar{i}$, $\bar{j}$, $\bar{k}$ vektorlar, birlik vektorlar, ya’ni $|\bar{i}|=1$; $|\bar{j}|=1$, $|\bar{k}|=1$

$\bar{a}$ vektor qanday bo‘lishidan qat’iy nazar uni har doim $\bar{i}$, $\bar{j}$, $\bar{k}$ bazislar bo‘yicha yoyish mumkin, ya’ni $\bar{a}=x_1\bar{i}+y_1\bar{j}+z_1\bar{k}$

Bu erda x_1, y_1, z_1 - $\bar{a}$ vektorning koordinatalari.

4-misol. $\bar{a}=\bar{i}+3\bar{j}-\bar{k}$ va $\bar{b}=2\bar{i}+\bar{j}+4\bar{k}$ vektorlar berilgan $2\bar{a}+3\bar{b}$ vektorlar yig‘indisini toping.

Yechish. $\bar{a}$ koordinatalari , $\bar{a}=(1;3;-2)$ xuddi shuningdek $\bar{b}=(2;1;4)$. Endi $2\bar{a}$ va $3\bar{b}$ vektorlarni aniqlaymiz.

$$2\bar{a}=(2;\ 6;\ -2);\quad 3\bar{b}=(6;\ 3;\ 12)$$

Demak, $2\bar{a}+3\bar{b}$=(2+6; 6+3; -2+12) =(8; 9; 10).

5-misol. $\bar{a}=(4,2,0)$ vektorni $\bar{p}=(1,-1,2), \bar{q}=(2,2,-1)$ va $\bar{r}=(3,7,-7)$ vektorlar bo‘yicha yoying.

Yechish. $\bar{a}$ vektorni $\bar{p},\bar{q}$ va $\bar{r}$ vektorlar bo‘yicha yoyish, $\bar{a}$ vektorni chiziqli kombinatsiya ko‘rinishida ifodalash demakdir.

$$\bar{a}=c_1\bar{p}+c_2\bar{q}+c_3\bar{r}\ ,$$

bu yerda c_1, c_2 va c_3 - topilishi kerak bo‘lgan sonlar.

Koordinata ko‘rinishida bu quyidagicha bo‘ladi.

$$4i + 2j + 0 \cdot k = (c_1 + 2c_2 + 3c_3)i + (-c_1 + 2c_2 + 7c_3)j + (2c_1 - c_2 - 7c_3)k$$

Natijada quyidagi tenglamalar tizimini hosil qilamiz.

$$\begin{cases} c_1 + 2c_2 + 3c_3 = 4 \\ -c_1 + 2c_2 + 7c_3 = 2 \\ 2c_1 - c_2 - 7c_3 = 0 \end{cases}$$

Buni yechib, $c_1 = 3; c_2 = -1; c_3 = 1$ ekanligini topamiz. Demak, $\bar{a} = 3\bar{p} - \bar{q} + \bar{r}$.

6-misol. $\bar{a} = (6;-2;-3)$ vektorning birlik vektorini toping.

Yechish. Birlik vektorni quyidagicha yozish mumkin.

$$\overline{a^0} = \bar{i}\cos\alpha + \bar{j}\cos\beta + \bar{k}\cos\gamma$$

$\cos\alpha, \cos\beta, \cos\gamma$ larni topamiz

$$\cos\alpha = \frac{\overline{a_x}}{|\bar{a}|}; \ \cos\beta = \frac{\overline{a_y}}{|\bar{a}|}; \ \cos\gamma = \frac{\overline{a_z}}{|\bar{a}|}$$

$$|\bar{\alpha}| = \sqrt{(6^2 + (-2)^2 + (-3)^2} = \sqrt{49} = 7$$

Bundan $\cos\alpha$=6/7; $\cos\beta$=-2/7; $\cos\gamma$=-3/7.

Demak, $\overline{a^0} = (\frac{6}{7}; -\frac{2}{7}; -\frac{3}{7})$.

2.3. Ikki vektorning skalyar ko‘paytmasi

$\bar{a}$ va $\bar{b}$ vektorlarning skalyar ko‘paytmasi deb, bu vektorlar uzunliklari ko‘paytmasi bilan ular orasidagi burchak kosinusining ko‘paytmasiga aytiladi va $(\bar{a}, \bar{b})$ shaklda belgilanadi.

$$(\bar{a}, \bar{b}) = |\bar{a}| \cdot |\bar{b}| \cos\varphi$$

$\bar{a}$ va $\bar{b}$ vektorlarning skalyar ko‘paytmasini quyidagicha ham yozish mumkin.

$(\bar{a}, \bar{b}) = |\bar{a}| \cdot np_{\bar{a}}\bar{b}$ yoki $(\bar{a}, \bar{b}) = |\bar{a}| \cdot np_{\bar{b}}\bar{a}$

Agar $\bar{a}$ va $\bar{b}$ vektorlar koordinatalari bilan berilgan bo‘lsa, ya’ni $\bar{a}=(x_1,y_1,z_1)$ va $\bar{b}=(x_2,y_2,z_2)$, u holda ularning skalyar ko‘paytmasi quyidagi formula bilan hisoblanadi. $(\bar{a},\bar{b})=(x_1\cdot x_2;y_1\cdot y_2;z_1\cdot z_2)$

Bundan vektorlarning **perpendikulyarlik** sharti kelib chiqadi, ya’ni $x_1\cdot x_2+y_1\cdot y_2+z_1\cdot z_2=0$

Koordinatalari bilan berilgan $\bar{a}=(x_1,y_1,z_1)$ va $\bar{b}=(x_2,y_2,z_2)$ vektorlar orasidagi φ burchak quyidagicha aniqlanadi:

$$\cos\varphi=\frac{(\overline{a},\overline{b})}{|\bar{a}|\cdot|\bar{b}|}$$

yoki koordinatalar shaklida

$$\cos\varphi=\frac{x_1\cdot x_2+y_1\cdot y_2+z_1\cdot z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_2^2+y_2^2+z_2^2}}$$

7-misol. Agar $\bar{p}=\bar{a}-\bar{b};\bar{q}=\bar{a}+2\bar{b};|\bar{a}|=1;|\bar{b}|=3;(\bar{a}\,\hat{}\,\bar{b})=\frac{2}{3}\pi$ bo‘lsa, $\bar{p}+2\bar{q}$ vektorning uzunligini toping.

Yechish. Vektorning moduli ta’rifiga ko‘ra: $|\bar{p}+2\bar{q}|=\sqrt{(\bar{p}+2\bar{q})^2}$. ($\bar{p}+2\bar{q}$)^2 ni hisoblaymiz.

$$(\bar{p}+2\bar{q})^2=(\bar{a}-\bar{b}+2\bar{a}+4\bar{b})^2=$$

$$9(\overline{a^2}+2\overline{ab}+\overline{b^2})=9(1+2\cdot3\cos\frac{2}{3}\pi+9)=63$$

Bundan $|\bar{p}+2\bar{q}|=\sqrt{63}=3\sqrt{7}$.

8-misol. $\bar{x}=(2,1,-2)$ vektorga kollinear va $(\bar{x}\cdot\bar{a})=27$ shartni qanoatlantiruvchi $\bar{a}$ vektorni toping.

Yechish.

Kollinearlik shartidan foydalanib $\overline{a}$ vektorni quyidagicha yozish mumkin. $\overline{a} = \lambda \overline{x}$, bu erda λ - noma'lum ko'paytuvchi. $\overline{a}$ vektorni topish uchun quyidagi shartdan foydalanamiz: $(\overline{x}\overline{a}) = \lambda \overline{a}^2 = \lambda(4+1+4) = 9\lambda = 27$.

Bundan $\lambda = 3$ va $\overline{x} = 3\overline{a} = (6,3,-6)$.

9-misol. Agar $\overline{a}$ =(1,-3,4) , $\overline{b}$ =(3,-4,2) va $\overline{c}$ =(-1,1,4) bo'lsa, $\overline{a}$ vektorning $\overline{b} + \overline{c}$ vektorga proeksiyasini hisoblang.

Yechish. Quyidagi formuladan foydalanamiz:

$$Пр_{\overline{b}+\overline{c}}\overline{a} = \frac{\overline{a}(\overline{b}+\overline{c})}{|\overline{b}+\overline{c}|}$$

$\overline{a}(\overline{b}+\overline{c})$ va $\overline{b}+\overline{c}$ ifodalarni hisoblaymiz.

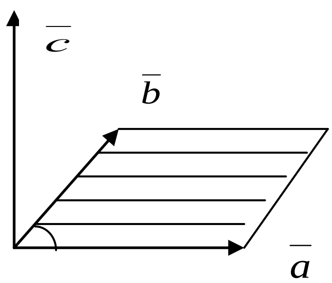
$$\overline{a}(\overline{b}+\overline{c}) = 1(3-1) - 3(-4+1) + 4(2+4) = 35$$

$$|\overline{b}+\overline{c}| = \sqrt{(3-1)^2 + (-4+1)^2 + (2+4)^2} = 7$$

Bundan $Пр_{\overline{b}+\overline{c}}\overline{a} = 5$

2.4. Ikki vektorningvektor ko'paytmasi

Ikki $\overline{a}$ va $\overline{b}$ vektorning vektor ko'paytmasi deb Shunday $\overline{c}$ vektorga aytiladki, bu vektor $\overline{a}$ va $\overline{b}$ vektorlarga pernpendikulyar bo'lib, uning moduli $\overline{a}$ va $\overline{b}$ vektorlardan yasalgan parallelogramm yuziga teng, yo'nalishi esa $\overline{c}$ uchidan qaraganda $\overline{c}$ vektor atrofida $\overline{a}$ vektordan $\overline{b}$ vektorga eng kichik burchak bilan aylanishi soat strelkasiga teskari bo'lishi kerak.



$\overline{a}$ vektor bilan va $\overline{b}$ vektorning vektor ko'paytmasi $\overline{a}\times\overline{b}$ yoki $[\overline{a}\cdot\overline{b}]$ shaklida yoziladi va quyidagicha belgilanadi.

$\bar{c} = [\bar{a} \cdot \bar{b}]$. Bu vektor uzunligi $\bar{a}$ va $\bar{b}$ vektorlardan yasalgan parallelogrammning yuziga teng; ya’ni

$\bar{c} = \left|[\bar{a} \cdot \bar{b}]\right| = |\bar{a}| \cdot |\bar{b}| \cdot \sin(\bar{a}, \hat{\ } \bar{b})$

Vektor ko‘paytmaning xossalari:

1. Vektor ko‘paytmadagi ko‘paytuvchilar o‘rnini almashtirsa, vektor ko‘paytma (-1) ga ko‘payadi.

$[\bar{a} \cdot \bar{b}] = -[\bar{b} \cdot \bar{a}]$

2. Skalyar ko‘paytuvchiga nisbatan vektor ko‘paytma gruppalash qonuniga bo‘ysunadi, ya’ni :

$[\lambda a, b] = [\bar{a}, \lambda \bar{b}] = \lambda [\bar{a} \cdot \bar{b}]$

Proeksiyalari bilan berilgan vektorlarning vektor ko‘paytmasi

$\bar{a}(x_1, y_1, z_1)$ va $\bar{b}(x_2, y_2, z_2)$ vektorlar berilgan bo‘lsin. Bu vektorlarning vektor ko‘paytmasi quyidagicha bo‘ladi.

$$[\bar{a} \cdot \bar{b}] = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

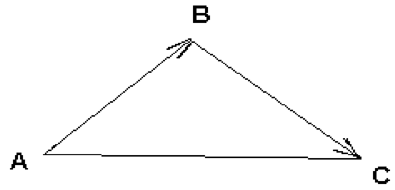
Bu tenglamadan $[\bar{a} \cdot \bar{b}]$ vektor ko‘paytmani tasvirlovchi vektorlarning koordinata o‘qlaridagi proeksiyalari

$y_1 z_2 - z_1 y_2;\ z_1 x_2 - x_1 z_2;\ x_1 y_2 - y_1 x_2;$

bo‘lishini ko‘ramiz.

Agar $\bar{a}$ va $\bar{b}$ vektorlar kollienar(bir-biriga parallel) bo‘lsa, ularning mos proyeksiyalari proporsional bo‘ladi, ya’ni $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$.

10-misol. Agar A(1,1,1), B(2,0,1) va C (1,2,-1) bo‘lsa, ABC uchburchakning yuzini hisoblang (1.rasm).



Yechish. Vektor ko‘paytmaning moduli son jihatdan, tomonlari shu vektorlardan qurilgan uchburchak yuzining ikkilanganiga teng:

$$S=\frac{1}{2}\left|\left[\bar{a}\bar{b}\right]\right|$$

$\bar{a}=\overline{AB}=(-1,1,0)$ va $\bar{b}=\overline{AC}=(0,1,-2)$ vektorlarni kiritamiz.

Bu vektorlarning vektor ko‘paytmasi.

$$\left[\bar{a}\bar{b}\right]=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -1 & 0\\ 0 & 1 & -2\end{vmatrix}=2\bar{i}+2\bar{j}+\bar{k}$$

$$\left|\bar{a}\bar{b}\right|=\sqrt{4+4+1}=3$$

Bundan S=1,5 kv.bir.

11-misol. Agar $|\bar{x}|=\sqrt{6}$ bo‘lsa, $\bar{a}=(1,1,1)$ va $\bar{b}=(2,0,1)$ vektorlarga perpendikulyar va OX o‘qi bilan o‘tmas burchak hosil qiluvchi $\bar{x}$ vektorni va $\bar{c}=\left[\bar{a}\bar{b}\right]$ vektorni toping.

Yechish. $\bar{c}$ vektorni kiritamiz

$$\bar{c}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 1 & 1\\ 2 & 0 & 1\end{vmatrix}=\bar{i}+\bar{j}-2\bar{k}$$

$\bar{x}$ vektor $\bar{a}$ i $\bar{b}$ vektorlarga perpendikulyar bo‘lsa, u holda $\bar{c}$ vektorga kollinear bo‘ladi. Bundan kelib chiqadiki, $\bar{x}=\lambda\bar{c}=(\lambda,\lambda,-2\lambda)$

$$|\bar{x}|=\sqrt{\lambda^2+\lambda^2+4\lambda^2}=\sqrt{6}\lambda=\sqrt{6}\rightarrow\sqrt{\lambda}=\pm1$$

$\bar{x}$ vektor OXo‘q bilan o‘tmas burchak tashkil qiladi, shuning uchun uning OX o‘qdagi proeksiyasi manfiy bo‘lishi kerak. Bundan $\lambda = -1$ va $\bar{x} = -c = (-1, -1, 2)$

12-misol. V (5,1,0) nuqtaga qo‘yilgan $\bar{F} = (1, -1, 1)$ kuch vektorining yo‘naltiruvchi kosinuslarini va shu kuchning A(3,2,-1) nuqtaga nisbatan momentini toping.

Yechish. kuch vektorining yo‘naltiruvchi kosinuslarini topamiz.

$$\cos\alpha = \frac{F_x}{|\bar{F}|} = \frac{1}{\sqrt{3}} \qquad \cos\beta = \frac{F_y}{|\bar{F}|} = -\frac{1}{\sqrt{3}} \qquad \cos\gamma = \frac{F_z}{|\bar{F}|} = \frac{1}{\sqrt{3}}$$

Kuch momenti $\overline{AB} = (2, -1, 1)$ va $\bar{F}$ vektorlarning vektor ko‘paytmasi kabi aniqlanadi.

$$\bar{m} = \left[\overline{AB}\bar{F}\right] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = -\bar{j} - \bar{k}$$

ya’ni $\bar{m} = (0, -1, -1)$.

2.5. Uch vektorning aralash ko‘paytmasi

Uchta $\bar{a}, \bar{b}, \bar{c}$ vektorlar berilgan bo‘lsin. $[\bar{a} \cdot \bar{b}]$ vektor ko‘paytma bilan $\bar{c}$ vektorni skalyar ko‘paytirish aralash ko‘paytma deyiladi va $[\bar{a} \cdot \bar{b}]\bar{c}$ yoki $\bar{a} \times \bar{b} \times \bar{c}$ yoki $(\overline{a \times b \times c})$ ko‘rinishda yoziladi.

Aralash ko‘paytmaning xossalari.

1. Ko‘paytmada ikki qo‘shni vektor o‘rni almashtirilsa, aralash ko‘paytma ishorasini almashtiradi.

$[\bar{a} \cdot \bar{b}] \cdot \bar{c} = -[\bar{c} \cdot \bar{b}] \cdot \bar{a}$

$[\bar{a} \cdot \bar{b}] \cdot \bar{c} = -[\bar{a} \cdot \bar{c}] \cdot \bar{b}$ va h.k.

2. Agar $\bar{a}, \bar{b}, \bar{c}$ vektorlardan istalgan ikkitasi bir-biriga teng yoki parallel (kollienar) bo‘lsa, ularning aralash ko‘paytmasi nolga teng, xususiy holda

$$[\bar{a}\cdot\bar{a}]\cdot\bar{c} = [\bar{a}\cdot\bar{b}]\cdot\bar{a} = [\bar{b}\cdot\bar{a}]\cdot\bar{a} = 0$$

3.Agar $\bar{a}, \bar{b}, \bar{c}$ vektorlar komplanar (bir tekislikda yotuvchi) vektorlar bo‘lsa, ularning aralash ko‘paytmasi nolga teng.

Proyeksiyalari bilan berilgan vektorlarning aralash ko‘paytmasi.

$\bar{a}(x_1, y_1, z_1)$, $\bar{b}(x_2, y_2, z_2)$ va $\bar{c}(x_3, y_3, z_3)$ vektorlar berilgan bo‘lsin. Bu vektorlarning vektor ko‘paytmasi quyidagicha bo‘ladi.

$$[\bar{a}\cdot\bar{b}]\cdot\bar{c} = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

$\bar{a}, \bar{b}, \bar{c}$ vektorlar komplanar bo‘lishining zaruriy va etarli sharti.

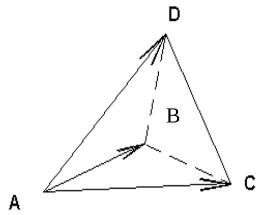
$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0$$

Tenglik bajarilishi bilan ifodalanadi.

13-misol. Agar A(2,3,1), V(4,1,-2), C(6,3,7) va D(-5,-4,8) nuqtalar piramidaning

uchlari bo‘lsa, (2-rasm) D uchidan ABC yoqqa tushirilgan balandlikning uzunligini toping.

Yechish: $\overline{AB} = (2, -2, -3)$; $\overline{AC} = (4, 0, 6)$; va $\overline{AD} = (-7, -7, 7)$



2-rasm

Vektorlarni topamiz. $\overline{AB}, \overline{AC}$ va $\overline{AD}$ vektorlarga qurilgan piramidaning hajmi, shu vektorlar aralash ko‘paytmasi modulining oltidan bir qismiga teng.

$$V = \frac{1}{6}\left| \overline{AB}\,\overline{AC}\,\overline{AD} \right| \text{ va } V = \frac{1}{3} S_{ABC} \cdot h$$

Bu erda $S_{ABC} = \frac{1}{2}\left|[\overline{AB} \cdot \overline{AC}]\right|$ bundan $h = \dfrac{\left|\overline{AB} \cdot \overline{AC} \cdot \overline{AD}\right|}{\left|[\overline{AB} \cdot \overline{AC}]\right|}$

Quyidagilarni hisoblaymiz

$$\overline{AB} \cdot \overline{AC} \cdot \overline{AD} = \begin{vmatrix} -7 & -7 & 7 \\ 2 & -2 & -3 \\ 4 & 0 & 6 \end{vmatrix} = 308$$

$$\left[\overline{AB} \cdot \overline{AC}\right] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -2 & 3 \\ 4 & 0 & 6 \end{vmatrix} = -12i + 24\bar{j} + 8\bar{k}$$

$$\left|\left[\overline{AB} \cdot \overline{AC}\right]\right| = \sqrt{12^2 + 24^2 + 8^2} = 28$$

Bu erdan $h = \frac{308}{28} = 11$

TOPSHIRIQ VARIANTLARI

Hisob topshiriqlarini qabul qilishda beriladigan nazariy savollar

1. Vektorning ta’rifi. Vektorlar ustida chiziqli amallar, bu amallarning xossalari.
2. Tekislikdagi vektorni berilgan ikkita vektor bo‘yicha yoyish.
3. Fazodagi vektorni berilgan uchta vektor bo‘yicha yoyish.
4. Vektorning o‘qqa proyeksiyasi. Proyeksiyaning xossasi.
5. Vektorni birlik vektorlar bo‘yicha yoyish. Vektorning koordinatalari va komponentlari.
6. Vektorlarning vektor va koordinatalar ko‘rinishidagi kollinearlik va komplanarlik shartlari.
7. Nuqtaning radius vektori. Vektorlarning moduli. Ikki nuqta orasidagi masofa.
8. Kesmani berilgan nisbatda bo‘lish formulasi
9. Vektorlarning skalyar ko‘paytmasi va uning fizik talqini. Skalyar ko‘paytmaning xossalari.
10. Vektorning vektorga proyeksiyasi. Vektorlar orasidagi burchak. Vektorlar perpendikulyarligining yetarli va zaruriy shartlari.
11. Koordinatalari bilan berilgan vektorlarning skalyar ko‘paytmasi.
12. Ikki vektorning vektor ko‘paytmasi va uning fizik talqini.
13. Koordinatalari bilan berilgan vektorlarning vektor ko‘paytmasi.
14. Vektor ko‘paytmaning geometrik qo‘llanilishi.
15. Vektor ko‘paytmaning xossalari.
16. Uchta vektorning koordinata ko‘rinishidagi aralash ko‘paytmasi.
17. Vektorlar komplanarligining zaruriy va yetarli sharti.
18. Koordinatalari bilan berilgan vektorlarning aralash ko‘paytmasi.
19. Aralash ko‘paytmaning xossalari.

1-topshiriq. $\bar{x}$ vektorning $\bar{p},\bar{q},\bar{r}$ vektorlar bo‘yicha yoyilmasini toping.

№	$\bar{x}$	$\bar{p}$	$\bar{q}$	$\bar{r}$
1.1.	(-2, 4, 7)	(0, 1, 2)	(1, 0, 1)	(-1, 2, 4)
1.2.	(6, 12, -1)	(1, 3, 0)	(2, -1, 1)	(0, -1, 2)
1.3.	(1, -4, 4)	(2, 1, -1)	(0, 3, 2)	(1, -1, 1)
1.4.	(-9, 5, 5)	(4, 1, 1)	(2, 0, -3)	(-1, 2, 1)
1.5.	(-5, -5, 5)	(-2, 0, 1)	(1, 3, -1)	(0, 4, 1)
1.6.	(13, 2, 7)	(5, 1, 0)	(2, -1, 3)	(1, 0, -1)
1.7.	(-19, -1, 7)	(0, 1, 1)	(-2, 0, 1)	(3, 1, 0)
1.8.	(3, -3, 4)	(I, 0, 2)	(0, 1, 1)	(2, -1, 4)
1.9.	(2, 2, -1)	(3, II, 0)	(-1, 2, 1)	(-1, 0, 2)
1.10.	(-1, 7, -4)	(-1, 2, 1)	(2, 0, 3)	(1, 1, -1)
1.11.	(6, 5, -14)	(1, 1, 4)	(0, -3, 2)	(2, 1, -1)
1.12.	(6, -1, 7)	(1, -2, 0)	(-1. 1, 3)	(1, 0, 4)
1.13.	(5, -15, 0)	(1, 0, 5)	(-1, 3, 2)	(0, -1, 1)
1.14.	(2, -1, 11)	(1, 1, 0)	(0, 1, -2)	(1, 0, 8)
1.15.	(11, 5, -3)	(1, 0, 2)	(-1, 0, 1)	(2, 5, -3)
1.16.	(8, 0, 5)	(2, 0, 1)	(1, 1, 0)	(4, 1, 2)
1.17.	(3, 1, 8)	(0, 1, 3)	(1, 2, -1)	(2, 0, -1)
1.18.	(8, 1, 12)	(1, 2, -1)	(3, 0, 2)	(-1, 1, 1)
1.19.	(-9, -8, -3)	(1, 4, 1)	(-3, 2, 1)	(1, -1, 2)
1.20.	(-5, 9, -13)	(0, 1, -2)	(3, -1, 1)	(4, 1, 0)
1.21.	(-15, 5, 6)	(0, 5, 1)	(3, 2, -1)	(-1, 1, 0)
1.22.	(8, 9, 4)	(1, 0, 1)	(0, -2, 1)	(1, 3, 0)
1.23.	(23, -14, -30)	(2, 1, 0)	(1, -1, 0)	(-3, 2, 5)
1.24.	(3, 1, 3)	(2, 1, 0)	(1, 0, 1)	(4, 2, 1)
1.25.	(-1, 7, 0)	(0, 3, 1)	(1, -1, 2)	(2, -1, 0)
1.26.	(11, -1, 4)	(1, -1, 2)	(3, 2, 0)	(-1, 1, 0)
1.27.	(-13, 2, 18)	(1, 1, 4)	(-3, 0, 2)	(1, 2, -1)
1.28.	(0, -8, 9)	(0, -2, 1)	(3, 1, -1)	(4, 0, 1)
1.29.	(8, -7, -13)	(0, 1, 5)	(3, -1, 2)	(-1, 0, 1)
1.30.	(2, 7, 5)	(1, 0, 1)	(1, -2, 0)	(0, 3, 1)

2-topshiriq. $\bar{a}$ ва $\bar{b}$ vektorlarga qurilgan $\overline{c_1}$ va $\overline{c_2}$ vektorlarning o‘zaro kollinearligini tekshiring.

№	$\bar{a}$	$\bar{b}$	$\overline{c_1}$	$\overline{c_2}$
2.1.	(1, -2, 3)	(3, 0, -1)	$2\bar{a}+4\bar{b}$	$3\bar{b}-\bar{a}$
2.2.	(1, 0, -1)	(-2, 3, 5)	$\bar{a}+2\bar{b}$	$3\bar{a}-\bar{b}$
2.3.	(-2, 4, 1)	(1, -2, 7)	$5\bar{a}+3\bar{b}$	$2\bar{a}-\bar{b}$
2.4.	(1, 2, -3)	(2, -1, -1)	$4\bar{a}+3\bar{b}$	$8\bar{a}-\bar{b}$
2.5.	(3, 5, 4)	(5, 9, 7)	$2\bar{a}+\bar{b}$	$3\bar{a}-2\bar{b}$
2.6.	(1, 4, -2)	(1, 1, -1)	$\bar{a}+\bar{b}$	$4\bar{a}+2\bar{b}$
2.7.	(1, -2, 5)	(3, -1, 0)	$4\bar{a}-2\bar{b}$	$\bar{b}-2\bar{a}$
2.8.	(3, 4, -1)	(2, -1, 1)	$6\bar{a}-3\bar{b}$	$\bar{b}-2\bar{a}$
2.9.	(2, -3, -2)	(1, 0, 5)	$3\bar{a}+9\bar{b}$	$\bar{a}-3\bar{b}$
2.10.	(-1, 4, 2)	(3, -2, 6)	$2\bar{a}-\bar{b}$	$3\bar{b}-6\bar{a}$
2.11.	(5, 0, -1)	(7, 2, 3)	$2\bar{a}-\bar{b}$	$3\bar{b}-6\bar{a}$
2.12.	(0, 3, -2)	(1, -2, 1)	$5\bar{a}-2\bar{b}$	$3\bar{a}+5\bar{b}$
2.13.	(-2, 7, -1)	(-3, 5, 2)	$2\bar{a}+3\bar{b}$	$3\bar{a}+2\bar{b}$
2.14.	(3, 7, 0)	(1, -3, 4)	$4\bar{a}-2\bar{b}$	$\bar{b}-2\bar{a}$
2.15.	(-1, 2, -1)	(2, -7, 1)	$6\bar{a}-2\bar{b}$	$\bar{b}-3\bar{a}$
2.16.	(7, 9, -2)	(5, 4, 3)	$4\bar{a}-\bar{b}$	$4\bar{b}-\bar{a}$
2.17.	(5, 0, -2)	(6, 4, 3)	$5\bar{a}-3\bar{b}$	$6\bar{b}-10\bar{a}$
2.18.	(8, 3, -1)	(4, 1, 3)	$2\bar{a}-\bar{b}$	$2\bar{b}-4\bar{a}$
2.19.	(3, -1, 6)	(5, 7, 10)	$4\bar{a}-2\bar{b}$	$\bar{a}-2\bar{b}$
2.20.	(1, -2, 4)	(7, 3, 5)	$6\bar{a}-3\bar{b}$	$\bar{b}-2\bar{a}$
2.21.	(3, 7, 0)	(4, 6, -1)	$3\bar{a}+2\bar{b}$	$5\bar{a}-7\bar{b}$
2.22.	(2, -1, 4)	(3, -7, -6)	$2\bar{a}-3\bar{b}$	$3\bar{a}-2\bar{b}$
2.23.	(5, -1, -2)	(6, 0, 7)	$3\bar{a}-2\bar{b}$	$4\bar{b}-6\bar{a}$
2.24.	(-9, 5, 3)	(7, 1, -2)	$2\bar{a}-\bar{b}$	$3\bar{a}+5\bar{b}$
2.25.	(4, 2, 9)	(0, -1, 3)	$4\bar{b}-3\bar{a}$	$4\bar{a}-3\bar{b}$
2.26.	(2, -1, 6)	(-1, 3, 8)	$5\bar{a}-2\bar{b}$	$2\bar{a}-5\bar{b}$
2.27.	(5, 0, 8)	(-3, 1, 7)	$3\bar{a}-4\bar{b}$	$12\bar{b}-9\bar{a}$
2.28.	(-1, 3, 4)	(2, -1, 0)	$6\bar{a}-2\bar{b}$	$\bar{b}-3\bar{a}$
2.29.	(4, 2, -7)	(5, 0, -3)	$\bar{a}-3\bar{b}$	$6\bar{b}-2\bar{a}$
2.30.	(2, 0, -5)	(1, -3, 4)	$2\bar{a}-5\bar{b}$	$5\bar{a}-2\bar{b}$

3-topshiriq. $\overline{AB}$ va $\overline{AC}$ vektorlar orasidagi burchak kosinusini toping.

№	*A*	*B*	*C*
3.1.	(6, 5, 1)	(0, 1, 2)	(2, 1, 0)
3.2.	(5, 4, 2)	(1, 2, 3)	(3, 2, 1)
3.3.	(2, 0, 4)	(1, 1, 1)	(3, 2, 1)
3.4.	(1, 2, 3)	(2, -1, 0)	(3, 2, 1)
3.5.	(1, -1, 2)	(5, -6, 2)	(2, 3, -1)
3.6.	(3, -3, 1)	(-3, -2, 0)	(5, 0, 2)
3.7.	(4, 2, 1)	(0, 4, 5)	(1, 2, 7)
3.8.	(1, 0, 2)	(2, 4, 3)	(1, 7, 1)
3.9.	(5, -1, 3)	(2, 0, 1)	(3, 1, -1)
3.10.	(0, 8, 1)	(2, 1, 1)	(-1, 4, 5)
3.11.	(1, 0, 4)	(0, 2, 3)	(-1, 1, 0)
3.12.	(2, 3, 4)	(3, 4, 5)	(-4, 5, 6)
3.13.	(1, -2, 3)	(0, -1, 2)	(3, -4, 5)
3.14.	(0, -3, 6)	(-12, -3, -3)	(-9, -3, -6)
3.15.	(3, 3, -1)	(5, 5, -2)	(4, 1, 1)
3.16.	(-1, 2, -3)	(3, 4, -6)	(1, 1, -1)
3.17.	(-4, -2, 0)	(-1, -2, 4)	(3, -2, 1)
3.18.	(5, 3, -1)	(5, 2, 0)	(6, 4, -1)
3.19.	(-3, -7, -6)	(0, -1, -2)	(2, 3, 0)
3.20.	(2, -4, 6)	(0, -2, 4)	(6, -8, 10)
3.21.	(0, 1, -2)	(3, 1, 2)	(4, 1, 1)
3.22.	(3, 3, -1)	(1, 5, -2)	(4, 1, 1)
3.23.	(2, 1, -1)	(6, -1, -4)	(4, 2, 1)
3.24.	(-1, -2, 1)	(-4, -2, 5)	(-8, -2, 2)
3.25.	(6, 2, -3)	(6, 3, -2)	(7, 3, -3)
3.26.	(0, 0, 4)	(-3, -6, 1)	(-5, -10, -1)
3.27.	(2, -8, -1)	(4, -6, 0)	(-2, -5, -1)
3.28.	(3, -6, 9)	(0, 3, 6)	(9, -12, 15)
3.29.	(0, 2, -4)	(8, 2, 2)	(6, 2, 4)
3.30.	(3, 3, -1)	(5, 1, -2)	(4, 1, 1)

4-topshiriq. $\overline{F}$ kuch vektorining yo‘naltiruvchi kosinuslarini aniqlang. A nuqtaga nisbatan V nuqtaga qo‘yilgan $\overline{F}$ kuch momentini toping.

№	$\overline{F}$	*B*	*A*
4.1.	(3, 3, 3)	(3, -1, 5)	(4,-2,3)
4.2.	(4, 4, 4)	(4, -2, 5)	(5,-3,3)
4.3.	(8, -8, 8)	(10, -8, 1)	(9,-7,3)
4.4.	(-2, 2, -2)	(11, -9, 1)	(10,-8,3)
4.5.	(5, 5, 5)	(5, -3, 5)	(6,-4,3)
4.6.	(-3, 3, -3)	(12, -10, 1)	(11,-9,3)
4.7.	(6, 6, 6)	(6, -4, 5)	(7,-5,3)
4.8.	(-4, 4, -4)	(13, -11, 1)	(12,-10,3)
4.9.	(7, 7, 7)	(7, -5, 5)	(8,-6,3)
4.10.	(-5, 5, -5)	(14, -12, 1)	(13, -11, 3)
4.11.	(-1, -1, 1)	(8, -6, -5)	(9, -7, 3)
4.12.	(3, 3, -3)	(0, 1, 2)	(2, -1, -2)
4.13.	(-2, -2, -2)	(9, -7, 5)	(10, -8, 3)
4.14.	(4, 4, -4)	(1, 0, 2)	(3, 2, -2)
4.15.	(-3, -3, -3)	(10, -8, 5)	(11, -9, 3)
4.16.	(5, 5, -5)	(2,-1,2)	(4, -3, 2)
4.17.	(-4, -4, -4)	(11,-9,5)	(12, -10, 3)
4.18.	(6, 6, -6)	(3,-2,2)	(5, -4, -2)
4.19.	(-5, -5, -5)	(12,-10,5)	(13, -11, 3)
4.20.	(7, 7, -7)	(4,-3,2)	(6, -5, -2)
4.21.	(3, -3, 3)	(5,-3,1)	(4, -2, 3)
4.22.	(8, 8, -8)	(5,-4,2)	(7, -6, -2)
4.23.	(4, -4, 4)	(6,-4,1)	(5, -4, 3)
4.24.	(-2, -2, 2)	(6,-5,2)	(8, -7, -2)
4.25.	(5, -5, 5)	(7,-5,1)	(6, -4, 3)
4.26.	(-3, -3, 3)	(7,-6,2)	(9, -8, 2)
4.27.	(6, -6, 6)	(8,-6,1)	(7, -5, 3)
4.28.	(-4, -4, 4)	(8,-7,2)	(10, -9, -2)
4.29.	(7, -7, 7)	(9,-7,1)	(8, -6, 3)
4.30.	(-5, -5, 5)	(9,-8,2)	(11, -10, 2)

5-t**opshiriq.** $\bar{a}$ va $\bar{b}$ vektorlaridan qurilgan parallelogrammning yuzasini hisoblang

№	$\bar{a}$	$\bar{b}$	$\lvert\bar{p}\rvert$	$\lvert\bar{q}\rvert$	$(\bar{p}\,\hat{}\,\bar{q})$
5.1.	$\bar{p}+2\bar{q}$	$3\bar{p}-\bar{q}$	1	2	$\frac{\pi}{6}$
5.2.	$3\bar{p}+\bar{q}$	$\bar{p}-2\bar{q}$	4	1	$\frac{\pi}{4}$
5.3.	$\bar{p}-3\bar{q}$	$\bar{p}+2\bar{q}$	$1/5$	1	$\frac{\pi}{2}$
5.4.	$3\bar{p}-2\bar{q}$	$\bar{p}+5\bar{q}$	4	$1/2$	$\frac{5\pi}{6}$
5.5.	$\bar{p}-2\bar{q}$	$2\bar{p}+\bar{q}$	2	3	$\frac{3\pi}{4}$
5.6.	$\bar{p}+3\bar{q}$	$\bar{p}-2\bar{q}$	2	3	$\frac{\pi}{3}$
.5.7.	$2\bar{p}-\bar{q}$	$\bar{p}+3\bar{q}$	3	2	$\frac{\pi}{2}$
5.8.	$4\bar{p}+\bar{q}$	$\bar{p}-\bar{q}$	7	2	$\frac{\pi}{4}$
5.9.	$\bar{p}-4\bar{q}$	$3\bar{p}+\bar{q}$	1	2	$\frac{\pi}{6}$
5.10.	$\bar{p}+4\bar{q}$	$2\bar{p}-\bar{q}$	7	2	$\frac{\pi}{3}$
5.11.	$3\bar{p}+2\bar{q}$	$\bar{p}-\bar{q}$	10	1	$\frac{\pi}{2}$
5.12.	$4\bar{p}-\bar{q}$	$\bar{p}+2\bar{q}$	5	4	$\frac{\pi}{4}$
5.13.	$2\bar{p}+3\bar{q}$	$\bar{p}-2\bar{q}$	6	7	$\frac{\pi}{3}$
5.14.	$3\bar{p}-\bar{q}$	$\bar{p}+2\bar{q}$	3	4	$\frac{\pi}{4}$
5.15.	$2\bar{p}+3\bar{q}$	$\bar{p}-2\bar{q}$	2	3	$\frac{\pi}{6}$

5.16.	$2\overline{p}-3\overline{q}$	$3\overline{p}+2\overline{q}$	4	1	$\frac{\pi}{6}$
5.17.	$3\overline{p}-2\overline{q}$	$2\overline{p}+3\overline{q}$	2	5	$\frac{\pi}{6}$
5.18.	$4\overline{p}-3\overline{q}$	$\overline{p}+2\overline{q}$	1	2	$\frac{\pi}{6}$
5.19.	$\overline{p}-\overline{q}$	$\overline{p}+\overline{q}$	2	5	$\frac{\pi}{6}$
5.20.	$5\overline{p}-\overline{q}$	$\overline{p}+5\overline{q}$	5	3	$\frac{\pi}{6}$
5.21.	$3\overline{p}-\overline{q}$	$\overline{p}+3\overline{q}$	2	$\sqrt{2}$	$\frac{\pi}{4}$
5.22.	$\overline{p}-4\overline{q}$	$\overline{p}+5\overline{q}$	$\sqrt{3}$	2	$\frac{\pi}{6}$
5.23.	$5\overline{p}+\overline{q}$	$\overline{p}-3\overline{q}$	1	2	$\frac{\pi}{3}$
5.24.	$7\overline{p}-2\overline{q}$	$\overline{p}+3\overline{q}$	$1/2$	2	$\frac{\pi}{2}$
5.25.	$6\overline{p}-\overline{q}$	$\overline{p}+\overline{q}$	3	4	$\frac{\pi}{4}$
5.26.	$10\overline{p}+\overline{q}$	$3\overline{p}-2\overline{q}$	4	1	$\frac{\pi}{6}$
5.27.	$6\overline{p}-\overline{q}$	$3\overline{p}+2\overline{q}$	8	$1/2$	$\frac{\pi}{3}$
5.28.	$3\overline{p}+4\overline{q}$	$\overline{p}-\overline{q}$	2,5	2	$\frac{\pi}{2}$
5.29.	$7\overline{p}+\overline{q}$	$\overline{p}-3\overline{q}$	3	1	$\frac{3\pi}{4}$
5.30.	$\overline{p}+3\overline{q}$	$3\overline{p}-\overline{q}$	3	5	$\frac{2\pi}{3}$

6-topshiriq. $\bar{a}$, $\bar{b}$, va $\bar{c}$ vektorlarningkomplanarligini anqlang.

№	$\bar{a}$	$\bar{b}$	$\bar{c}$
6.1.	(2,3,1)	(-1,0,-1)	(2,2,2)
6.2.	(3,2,1)	(2,3,4)	(3,1,-1)
6.3.	(1,5,2)	(-1,1,-1)	(1,1,1)
6.4.	(1,-1,-3)	(3,2,1)	(2,3,4)
6.5.	(3,3,1)	(1,-2,1)	(1,1,1)
6.6.	(3,1,-1)	(-2,-1,0)	(5,2,-1)
6.7.	(4,3,1)	(1,-2,1)	(2,2,2)
6.8.	(4,3,1)	(6,7,4)	(2,0,-1)
6.9.	(3,2,1)	(1,-3,-7)	(1,2,3)
6.10.	(3,7,2)	(-2,0,-1)	(2,2,1)
6.11.	(1,-2,6)	(1,0,1)	(2,-6,17)
6.12.	(6,3,4)	(-1,-2,-1)	(2,1,2)
6.13.	(7,3,4)	(-1,-2,-1)	(4,2,4)
6.14.	(2,3,2)	(4,7,5)	(2,0,-1)
6.15.	(5,3,4)	(-1,0,-1)	(4,2,4)
6.16.	(3,10,5)	(-3,-2,-3)	(2,4,3)
6.17.	(-2,-4,-3)	(4,3,1)	(6,7,4)
6.18.	(3,1,-1)	(1,0,-1)	(8,3,-2)
6.19.	(4,2,2)	(-3,-3,-3)	(2,1,2)
6.20.	(4,1,2)	(9,2,5)	(1,1,-1)
6.21.	(5,3,4)	(4,3,3)	(9,5,8)
6.22.	(3,4,2)	(1,1,0)	(8,11,6)
6.23.	(4,-1,-6)	(1,-3,-7)	(2,-1,-4)
6.24.	(3,1,0)	(-5,-4,-5)	(4,2,4)
6.25.	(3,0,3)	(8,1,6)	(1,1,-1)
6.26.	(1,-1,4)	(1,0,3)	(1,-3,8)
6.27.	(6,3,4)	(-1,-2,-1)	(2,1,2)
6.28.	(4,1,1)	(-9,-4,-9)	(6,2,6)
6.29.	(-3,3,3)	(-4,7,6)	(3,0,-1)
6.30.	(-7,10,-5)	(0,-2,-1)	(-2,4,-1)

7- topshiriq Uchlari A, B, C,va D nuqtalarda bulgan piramidaning hajmini va D uchidan ABC yoqqa tushirilgan balandligini toping.

№	A	B	C	D
7.1.	(0,1,2)	(2,1,7)	(2,7,4)	(0,0,4)
7.2.	(1,2,3)	(2,8,-4)	(0,5,4)	(2,9,4)
7.3.	(1,1,1)	(2,4,-2)	(2,0,2)	(0,1,-1)
7.4.	(1,-1,1)	(0,2,3)	(1,-1,0)	(0,2,2)
7.5.	(2,1,3)	(4,-2,0)	(1,3,-3)	(7,5,2)
7.6.	(-2,0,4)	(1,3,-1)	(4,-1,3)	(2,7,3)
7.7.	(1,2,3)	(0,0,0)	(1,4,3)	(1,8,-1)
7.8.	(-1,2,0)	(1,0,3)	(0,2,2)	(1,8,3)
7.9.	(2,-1,1)	(3,3,2)	(2,1,0)	(4,1,-3)
7.10.	(2,1,-1)	(-3,1,2)	(0,1,2)	(-1,8,3)
7.11.	(-2,1,1)	(5,5,4)	(3,2,-1)	(4,1,3)
7.12.	(0,1,-1)	(3,-1,5)	(1,0,4)	(3,5,7)
7.13.	(1,1,2)	(-1,1,3)	(2,-2,4)	(-1,0,-2)
7.14.	(2,3,1)	(4,1,-2)	(6,3,7)	(7,5,-3)
7.15.	(1,1,-1)	(2,3,1)	(3,2,1)	(5,9,-8)
7.16.	(1,5,-7)	(-3,5,3)	(-2,7,3)	(-4,8,-12)
7.17.	(-3,4,-7)	(1,5,-4)	(-6,-2,0)	(2,5,4)
7.18.	(-1,2,-3)	(4,-1,0)	(2,1,-2)	(3,4,5)
7.19.	(4,-1,3)	(-2,1,0)	(0,-5,1)	(3,2,-6)
7.20.	(1,-1,1)	(-2,0,3)	(2,1,-1)	(2,-2,-4)
7.21.	(1,2,0)	(1,-1,2)	(0,1,-1)	(-3,0,1)
7.22.	(1,0,2)	(1,2,-1)	(2,-2,1)	(2,1,0)
7.23.	(1,2,-3)	(1,0,1)	(-2,-1,6)	(0,-5,-4)
7.24.	(3,10,-1)	(-2,3,-5)	(-6,0,-3)	(1,-1,2)
7.25.	(-1,2,4)	(-1,-2,-4)	(3,0,-1)	(7,-3,1)
7.26.	(0,-3,1)	(-4,1,2)	(2,-1,5)	(3,1,-4)
7.27.	(1,3,0)	(4,-1,2)	(3,0,1)	(-4,3,5)
7.28.	(-2,-1,-1)	(0,3,2)	(3,1,-4)	(-4,7,3)
7.29.	(-3,-5,6)	(2,1,-4)	(0,-3,-1)	(-5,2,-8)
7.30.	(2,-4,-3)	(5,-6,0)	(-1,3,-3)	(-10,-8,7)

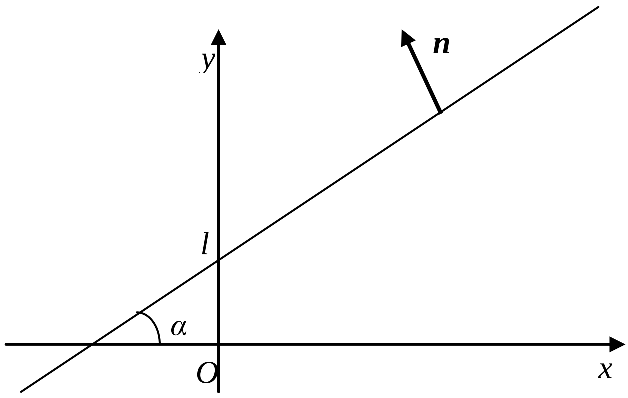
III. ANALITIK GEOMETRIYA

3.1. Tekislikda to‘g‘ri chiziq

1^0. *To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi:*

$$y = kx + l \tag{1}$$

bu erda $k = tg\alpha$ to‘g‘ri chiziqning abssissa o‘qi musbat yo‘nalishi bilan tashkil etgan burchak tangensi, $l-$ ordinata o‘qidan koordinatalar boshiga nisbatan qancha masofadan kesib o‘tishini ifodalovchi son(1.1-rasm).



Agar l=0 bo‘lsa, y= kx to‘gri chiziq koordinatalar boshidan o‘tadi; agar x=0 bo‘lsa, y= l abssissa o‘qiga nisbatan parallel o‘tuvchi to‘g‘ri chiziq; $u = 0$ bo‘lsa, x=a ordinata o‘qiga nisbatan parallel to‘g‘ri chiziq tenglamasi bo‘ladi.

2^0. *To‘g‘ri chiziqning umumiy tenglamasi:*

$$Ax + By + C = 0 \tag{2}$$

bu erda A,V o‘zgarmas koeffitsentlar bo‘lib, to‘g‘ri chiziqqa perpendikulyar yo‘nalgan $\vec{n} = \{A, B\}$ vektorning koordinatalari; C – ozod had.

Xususiy holda:

a) $A \neq 0, \quad B \neq 0, \quad C = 0$ bo‘lsa, $Ax + By = 0$ koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq tenglamasi;

b) $A = 0, \quad B \neq 0, \quad C \neq 0$ bo‘lsa, $y = -\frac{C}{B} = l$ –abssissa o‘qiga parallel to‘g‘ri chiziq tenglamasi;

v) $A \neq 0, \quad B = 0, \quad C \neq 0$ bo‘lsa, $x = -\frac{C}{A} = k$ – ordinata o‘qiga parrallel to‘g‘ri chiziq tenglamasi;

3^0. *To‘g‘ri chiziqning kesmalar bo‘yicha tenglamasi:*

$$\frac{x}{a} + \frac{y}{b} = 1 \tag{3}$$

bu yerda *a,b* – to‘g‘ri chiziqning mos ravishda Ox va Oy o‘qlarida ajratgan kesmalari.

4^0. *To‘g‘ri chiziqning normal tenglamasi:*

$$x\cos\alpha + y\sin\alpha - p = 0 \tag{4}$$

bu yerda $cos\,\alpha, sin\,\alpha$ to‘g‘ri chiziqqa perpendikulyar yo‘nalgan vektorning koordinatalari, ya’ni $\vec{n} = \{\cos\alpha, \sin\alpha\}$ moduli birga teng $|\vec{n}| = \sqrt{cos^2\alpha + sin^2\alpha}$ bo‘lgan vektor; *p* – koordinatalar boshidan to‘g‘ri chiziqgacha bo‘lgan masofa.

5^0. Agar to‘g‘ri chiziq $M_1(x_1, y_1)$ nuqtadan o‘tsa, u holda;

$$y - y_1 = k(x - x_1) \tag{5}$$

to‘g‘ri chiziqlar dastasini tenglamasi bo‘ladi.

6^0. *Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi:*

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \tag{6}$$

bu yerda x_1, y_1, x_2, y_2 berilgan nuqtaning koordinatadlari.

7^0. *Ikki to‘g‘ri chiziq orasidagi burchak*

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 k_2} \qquad (7)$$

formula bilan aniqlanadi.

Xususiy holda:

a) Agar $k_1 = k_2$ bo‘lsa, to‘g‘ri chiziqlar o‘zaro parallel;

b) Agar $k_1 \cdot k_2 = -1$ bo‘lsa, to‘g‘ri chiziqlar o‘zaro perpendikulyar bo‘ladi

8^0. AB *kesmani* λ *nisbatda bo‘luvchi nuqtaning koordinatalri*

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda}, \quad y = \frac{y_1 + \lambda y_2}{1 + \lambda}; \qquad (8)$$

formulasi orqali aniqlanadi;

Xususan, $\lambda = 1$ ga teng bo‘lganda, kesma o‘rtasining koordinatalari

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2} \qquad (9)$$

formulasi bilan aniqlanadi.

9^0. *Berilgan* $A(x_1, y_1)$ *nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa:*

$$d = |x_1 \cos\alpha + y_1 \sin\alpha - p| \text{ yoki } d = \left| \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}} \right|. \qquad (10)$$

10^0. U*ch burchak yuzini hisoblash formulasi:*

$$S = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad (11)$$

bu yerda $A(x_1, u_1)$, $V(x_2, u_2)$, $S(x_3, u_3)$ uchburchak uchining koordinatalari.

11^0. *Ikki* $A(x_1, u_1)$ *va* $V(x_2, u_2)$ *nuqta orasidagi masofa quyidagi formula bilan topiladi:*

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (12)$$

1-misol. Uchburchak uchining koordinatalari berilgan:

$$A(-1;4),\quad B(3;6)\quad\ C(5;-2)$$

Quyidagilarni toping:

1) *AC* va *AC* tomon tenglamalarini;

2) *AB* va *AC* tomonlar orasidagi burchakni;

3) *B* uchidan *AC* tomonga tushirilgan balandlik tenglamasini;

4) *C* uchidan *AC* tomonga tushirilgan balandlik tenglamasini;

5) *A* uchida *BC* tomonga bo‘lgan masofani;

6) *ABC* uchburchak yuzini.

Yechish:

1) Berilgan *A(-1;4) va B(3;6),* nuqtalardan o‘tuvchi AB tomon tenglamasini tuzamiz: buning uchun (6) formulasidan foydalanamiz,

ya’ni $\frac{x-x_1}{y_2-y_2}=\frac{y-y_1}{y_2-y_1}$

$$\frac{x+1}{3+1}=\frac{y-4}{6-4}\Rightarrow\frac{x+1}{4}=\frac{y-4}{2}\Rightarrow 2(x+1)=4(u-4)\ \text{yoki}\ 2(y-4)=x+1\Rightarrow$$

$$y-4=\frac{1}{2}x+\frac{1}{2}\Rightarrow y=\frac{1}{2}x+\frac{9}{2};\quad k_1=\frac{1}{2},\quad l_1=\frac{9}{2};$$

Xuddi shuningdek, *A(-1;4) va C(5;-2)* nuqtalardan o‘tuvchi AC tomon tenglamasini tuzamiz:

$$\frac{x+1}{5+1}=\frac{y-4}{-2-4}\Rightarrow\frac{x+1}{6}=\frac{y-4}{-6}\Rightarrow y-4=-x+1\Rightarrow y=-x+5$$

$$k_2=-1,\qquad l_2=3;$$

2) Ikki to‘g‘ri chiziq orasidagi burchakni topish formulasi (7) dan foydalanib, AB va AC tomonlar orasidagi burchakni topamiz:

$$AB:\quad y=\frac{1}{2}x+\frac{9}{2},\qquad k_1=\frac{1}{2}$$

$$AC:\quad y=-x+3,\qquad k_2=-1$$

$$tg\varphi=\frac{k_2-k_1}{1+k_1k_2}\Rightarrow tg\varphi=\frac{-1-\frac{1}{2}}{1-1\cdot\frac{1}{2}}\Rightarrow tg\varphi=\frac{-\frac{3}{2}}{\frac{1}{2}}$$

$$tg\varphi=-3;\quad \varphi=arctg(-3);$$

3) B uchidan AC tomonga tushirilgan balandlik tenglamasini tuzamiz: Buning uchun berilgan nuqtadan o‘tuvchi to‘g‘ri chiziq formulasi (5) dan foydalanamiz:

$$y-y_2=k(x-x_1)$$

B(3;6) nuqtadan o‘tuvchi to‘g‘ri chiziq $y-6=k(x-3)$ ko‘rinishiga ega bo‘ladi. Bu to‘g‘ri chiziq AC to‘g‘ri chiziqqa perpendikulyar bo‘lishi uchun 7° da ko‘rsatilgan perpendikulyarlik shartidan $k\cdot k_2=-1$ foydalanamiz:

$$AC:\quad y=-x+3,\quad k_2=-1;$$

shu $k_2=-1$ ni $k\cdot k_2=-1$ tenglikka qo‘ysak $k\cdot(-1)=-1$ dan $k=1$ ekanligi kelib chiqadi. $k=1$ ni $y-6=k(x-3)$ tenglikka qo‘yamiz $y-6=1(x-3)\Rightarrow y=x+3;$ Bu *B(3;6)* nuqtadan o‘tuvchi balandlik tenlamasi bo‘ladi.

4) C uchidan o‘tuvchi AB tomonga tushirilgan mediana tenglamasini tuzish uchun AB tomonni teng ikkiga bo‘luvchi nuqtaning koordinatalarini topishimiz kerak. Buning uchun (9) dan foydalanamiz: A(-1;4), B(3;6)

$$x=\frac{x_1+x_2}{2},\qquad x=\frac{-1+3}{2}=\frac{2}{2}=1$$

$$y=\frac{y_1+y_2}{2},\qquad y=\frac{4+6}{2}=\frac{10}{2}=5$$

D(1;5) nuqta AB kesma o‘rtasi.

Mediana C(5;-2) va D(1;5) nuqtalardan o‘tadi va ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi formulasidan foydalanamiz

$$CD: \quad \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1} \Rightarrow \frac{x-5}{1-5}=\frac{y+2}{5+2} \Rightarrow \frac{x-5}{-4}=\frac{y+2}{7}$$

$7(x-5)=-4(y+2) \Rightarrow 7x-35=-4y-8 \ \ yoki \ \ 4y=-7x+27$

$y=-\frac{7}{4}x+\frac{27}{4}$ mediana tenglamasi.

5) A uchidan BC tomongacha bo‘lgan masofani topish uchun avval BC tomon tenglamasini tuzamiz B(3;6), C(5;-2)

$$BC: \quad \frac{x-3}{5-3}=\frac{y-6}{-2-6} \Rightarrow \frac{x-3}{2}=\frac{y-6}{-8} \Rightarrow -8(x-3)=2(y-6)$$

$-8x+24=2y-12 \Rightarrow 2y+8x-36=0 \Rightarrow 4x+2y-18=0$

$4x+y-18=0$ VS tomon tenglamasi. *A=4, B=1, C=-18*

A nuqtadan BC tomongacha bo‘lgan masofani topish uchun (10) formulasidan foydalanamiz $d=\left|\frac{Ax_1+By_1+C}{\pm\sqrt{A^2+B^2}}\right|$ qiymatlarni o‘rniga qo‘yamiz

$d=\left|\frac{4\cdot(-1)+1\cdot 4-18}{\pm\sqrt{4^2+1^2}}\right|=\left|\frac{[-4+4-18}{\sqrt{17}}\right|=\frac{|-18|}{\sqrt{17}}=\frac{18}{\sqrt{17}}$

6)ABC uchburchak yuzini topish uchun (11) formuladan foydalanamiz: Bizga uchburchak o‘zining koordinatalari A(-1;4), B(3;6), C(5;-2) berilgan shu sababli mos koordinatalarni formulaga qo‘yamiz:

$$S=\pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\pm\begin{vmatrix} -1 & 4 & 1 \\ 3 & 6 & 1 \\ 5 & -2 & 1 \end{vmatrix}=\pm\frac{1}{2}(-6+20-6-30-$$

$$12-2)==\pm\frac{1}{2}(-36)=-\frac{1}{2}(-36)=18$$

ABC uchburchak yuzi S=18 kv.birlikga teng.

3.2. Tekislik

1°. *Tekislikning umumiy tenglamasi:*

$$Ax + By + Cz + D = 0 \quad (1)$$

ko‘rinishiga ega bo‘lib, A,B,C noma’lumlar oldidagi koeffitsiyentlar hamda shu tekislikka perpendikulyar bo‘lgan $\vec{n}\{A, B, C\}$ vektorning koordinatalaridir, D – esa ozod had:

Xususiy hollari:

a) $A \neq 0, B \neq 0, C \neq 0, D = 0$ ya’ni tenglama $Ax + By + Cz = 0$ ko‘rinishda bo‘lsa, tekislik koordinatalar boshidan o‘tadi;

b) $A \neq 0,\ B \neq 0,\ D \neq 0,\ C = 0$ bo‘lsa, ya’ni $Ax + By + D = 0$ Oxy tekislikda to‘g‘ri chiziq, fazoda esa Oz o‘qiga parallel tekislik;

v) $A \neq 0,\ C \neq 0,\ D \neq 0,\ B = 0$ bo‘lsa, ya’ni $Ax + Cz + D = 0$ tenglama Oxz tekislikda to‘g‘ri chiziq, fazoda esa Oy parallel tekislik;

g) $B \neq 0,\ C \neq 0,\ D \neq 0,\ A = 0$ bo‘lsa, ya’ni $By + Cz + D = 0$ tenglama Oyz tekislikda to‘g‘ri chiziq, fazoda esa Ox o‘qiga parallel tekislik;

d) $A \neq 0, D \neq 0,\ B = C = 0$ bo‘lsa, $Ax + D = 0$yoki $x = -\frac{D}{A} = a$ tenglama, *Ox* o‘qida nuqta, fazoda *Oyz* tekislikka parallel tekislik;

e) $B \neq 0,\ D \neq 0,\ A = C = 0$ bo‘lsa, $By + D = 0$ yoki $y = -\frac{D}{B} = b$ tenglama, *Oy* o‘qida nuqta, fazoda *Oxz* tekislikka parallel tekislik;

j) $C \neq 0,\ D \neq 0, A = B = 0$ bo‘lsa, $Cz + D = 0$ yoki $z = -\frac{D}{C}$ tenglama Oz o‘qida nuqta, fazoda Oxytekislikka parallel tekislik;

2°. *Tekislikning umumiy tenglamasidan kesmalar bo‘yicha tenglamasini keltirib chiqarish mumkin:*

$Ax + By + Cz + D = 0$ dan $A \neq 0, B \neq 0, C \neq 0, D \neq 0$;

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \qquad a = -\frac{D}{A},\ b = -\frac{D}{B},\ C = -\frac{D}{C} \qquad (2)$$

3°. *Ikki tekislik orasidagi burchak:*

Ikkita tekislik tenglamalari berilgan bo‘lsin:

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0, & \overrightarrow{n_1}(A_1, B_1, C_1) \\ A_2x + B_2y + C_2z + D_2 = 0, & \overrightarrow{n_2}(A_2, B_2, C_2) \end{cases}$$

tekisliklar orasidagi burchak ular perpendikulyar bo‘lgan normal vektorlar orasidagi burchak bilan o‘lchanadi, shu sabab bu vektorlarning skalyar ko‘paytmasi formulasidan foydalanamiz:

ya’ni $\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1| \cdot |\vec{n}_2| cos\varphi, \qquad \varphi = (\vec{n}_1,^\wedge \vec{n}_2)$

$$cos\varphi = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} \qquad (3)$$

ikki tekislik orasidagi burchak formulasi.

Xususiy holda:

a) Agar tekisliklar o‘zaro parallel bo‘lsa, $\varphi = 0$, ularning normal vektorlari kollinear bo‘ladi, ya’ni $\vec{n}_1 = \gamma\vec{n}_2$ tenglik o‘rinma bo‘ladi, bu yerdan

$$\gamma = \frac{\vec{n}_1}{\vec{n}_2}, \qquad \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (4)$$

ekanligi kelib chiqadi. (parallellik sharti)

b) Agar tekisliklar o‘zaro perpendikulyar bo‘lsa, $\varphi = 90°$ u holda (3) formuladan

$$A_1A_2 + B_1B_2 + C_1C_2 = 0 \qquad (5)$$

perpendikulyarlik shart kelib chiqadi, chunki $cos90° = 0$ ga teng.

4°. Agar fazodа $M_1(x_{1,}y_{1,}z_1)$ nuqta berilgan bo‘lsa, M_1 nuqtadan o‘tuvchi tenglik tenglamasi

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0 \qquad (6)$$

fomulasi bilan aniqlanadi.

5° Uchta nuqtadan o‘tuvchi tekislik tenglamasi, ya’ni $M_1(x_{1,}y_{1,}z_1)$, $M_2(x_{2,}y_{2,}z_2)$, $M_3(x_{3,}y_{3,}z_3)$ nuqtalar berilgan bo‘lsa,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-x_1 & z_3-z_1 \end{vmatrix}=0 \qquad (7)$$

formulasi bilan aniqlanadi.

6°. Tekislikning normal tenglamasi

$$cos\alpha \cdot x+cos\beta \cdot y+cos\gamma \cdot z-p=0 \qquad (8)$$

ko‘rinishiga ega va $cos^2\alpha+cos^2\beta+cos^2\gamma=1$ tenglik o‘rinli. Bu $cos\alpha, cos\beta, cos\gamma$ifodalar normal vektorning koordinatalari $\vec{n}(cos\alpha, cos\beta, cos\rho)$ bo‘lib, ular

$$cos\alpha=\frac{A}{\pm\sqrt{A^2+B^2+C^2}},\ cos\beta=\frac{B}{\pm\sqrt{A^2+B^2+C^2}},\ cos\gamma=\frac{C}{\pm\sqrt{A^2+B^2+C^2}}$$

tengliklar orqali aniqlanadi $p$esa $p=\frac{D}{\pm\sqrt{A^2+B^2+C^2}}$ga teng.

Demak ,

$$\frac{A}{\pm\sqrt{A^2+B^2+C^2}}x+\frac{B}{\pm\sqrt{A^2+B^2+C^2}}y+\frac{C}{\pm\sqrt{A^2+B^2+C^2}}z+\frac{D}{\pm\sqrt{A^2+B^2+C^2}}=0$$

tekislikning normal tenglamasi bo‘ladi.

7°*Nuqtadan tekislikgacha bo‘lgan masofa:*

$$d=|x_1cos\alpha+y_1cos\beta+z_1cos\rho| \qquad (9)$$

yoki

$$d=\left|\frac{Ax_1+By_1+Cz_1+D}{\pm\sqrt{A^2+B^2+C^2}}\right| \qquad (10)$$

formulasi bilan aniqlanadi.

Demak, berilgan $M_1(x_{1,}y_{1,}z_1)$ nuqtadan tekislikkacha bo‘lgan masofani topish uchun, tekislikning normal tenglamasidagi nuqtaning koordinatalarini qo‘yib hisoblash kerak.

2-misol. Fazoda piramida uchining koordinatalari berilgan: $A_1(3;2;-2),\ A_2(6;-1;7),\ A_3(1;3;5),\ A_4(-3;0;4),$

1) $A_1A_2A_3$ va $A_1A_2A_4$ tekislik tenglamalari tuzing;

2) $A_1A_2A_3$ va $A_1A_2A_4$ tekislik orasidagi burchakni toping;

3) A_4 nuqtadan o‘tib $A_1A_2A_3$ tekislikka parallel tekislik tenglamasini tuzing;

4) A_3nuqtadan $A_1\ A_2A_4$ tekislikkacha bo‘lgan masofani toping;

5) $A_1A_2A_3$ tekislik tenglamasini kesmalar bo‘yicha ifodalang;

Yechish:

1)$A_1A_2A_3$ va $A_1A_2A_4$ tomon tenglamalari tuzamiz:

$A_{1,}A_2A_3$ tomon tenglamasini tuzish uchun (1) formulasidan foydalanamiz:

$$\begin{vmatrix} x-3 & y-2 & z+2 \\ 6-3 & -1-2 & 7+2 \\ 1-3 & 3-2 & 5+2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-3 & y-2 & z+2 \\ 3 & -3 & 9 \\ -2 & 1 & 7 \end{vmatrix} = 0$$

$$-30(x-3) - 39(y-2) - 3(z+2) = 0$$

$$10(x-3) + 13(y-2) + 1(z+2) = 0 \quad yoki \quad 10x + 13y + z - 5$$

Xuddi shuningdek $A_1A_2A_4$ tomon tenglamasini tuzamiz:

$$\begin{vmatrix} x-3 & y-2 & z+2 \\ 6-3 & -1-2 & 7+2 \\ -3-3 & 0-2 & 4+2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-3 & y-2 & z+2 \\ 3 & -3 & 9 \\ -6 & -2 & 6 \end{vmatrix} = 0$$

$$0(x-3) - 72(y-2) + 24(z+2) = 0$$

$$3(y-2) + (z+2) = 0 \quad yoki \quad 3y + z - 4 = 0$$

2)$A_1A_2A_3$ va $A_1A_2A_4$ tekisliklar orasidagi burchakni topish uchun berilgan $\begin{cases} 10x + 13y + z - 54 = 0 \\ 0x + 3y + z - 4 = 9 \end{cases}$ tekisliklardan normal vektorlarni yozamiz $\vec{n}_1$(10;13;1), $\vec{n}_2$(0;3;1), va (3) formuladan foydalanib tekisliklar orasidagi burchakni aniqlaymiz:

$$cos\varphi = \frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}|\cdot|\overrightarrow{n_2}|} = \frac{10\cdot0+13\cdot3+1\cdot1}{\sqrt{10^2+13^2+1^2}\cdot\sqrt{0^2+3^2+1^2}} = \frac{40}{\sqrt{270}\cdot\sqrt{10}} = \frac{40}{30\sqrt{3}}$$

$$cos\varphi = \frac{4}{3\sqrt{3}}\ , \quad cos\varphi = \frac{4\sqrt{3}}{9}, \qquad \varphi = arccos\frac{4\sqrt{3}}{9}$$

3) A_4 nuqtadan o‘tib $A_1A_2A_3$ tekislikka parallel tekislik tenglamasini tuzish uchun A_4(-3:0:4) nuqtadan o‘tuvchi tekislik tenglamasini (6) formuladan foydalanib yozamiz:

$$A(x+3) + B(y-0) + C(z-4) = 0 \qquad (*)$$

bu tekislik $10x + 13y + z - 54 = 0$ tekislikka parallel bo‘lishi uchun (4) munosabatidan foydalanamiz, ya’ni parallelik sharti

$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ dan $\frac{A}{10} = \frac{B}{13} = \frac{C}{1} = \gamma$.

$\gamma \neq 0$, $A = 10\gamma$, $B = 13\gamma$, $C = \gamma$. A,V va S larning topilpgan qiymatlarini (*) tenglamaga qo‘yamiz:

$$10\gamma(x+3) + 13\gamma y + C\gamma(z-4) = 0$$

Bundan $10x + 13y + z + 26 = 0$.

4)A_3 nuqtadan $A_1A_2A_4$ tekislikgacha bo‘lgan masofani topish uchun (10) formuladan foydalanamiz:

$A_3(1;3;5)$ nuqta va $A_1A_2A_4$ tekislik tenglamasi $3y + z - 4 = 0$ ga asosan quyidagicha yozamiz:

$$d = \left|\frac{0\cdot1 + 3\cdot3 + 1\cdot5 - 4}{\pm\sqrt{0^2 + 3^2 + 1^2}}\right| = \left|\frac{10}{\sqrt{10}}\right| = \sqrt{10}$$

5)$A_1\ A_2A_3$ tekislik tenglamasini tekislikning kesmalar bo‘yicha tenglamali ko‘rinishda ifodalaymiz:

$$10x+13y+z-54=0 \quad \Rightarrow\ 10x+13y+z=54$$

$$\frac{10x}{54}+\frac{13y}{54}+\frac{z}{54}=1\ \Rightarrow \quad \frac{x}{\frac{54}{10}}+\frac{y}{\frac{54}{13}}+\frac{z}{54}=1 \quad \Rightarrow \quad a=\frac{54}{10},$$

$$b=\frac{54}{13},\ \ c=54$$

Xuddi shuningdek $A_1A_2A_4$ tekislik tenglamasini ham yozishimiz mumkin.

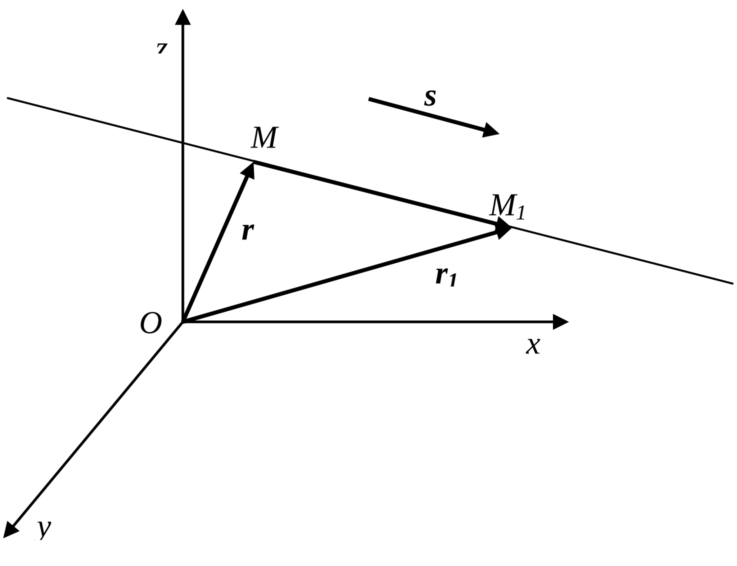
$$3y+z=4\ \Rightarrow \quad \frac{y}{\frac{4}{3}}+\frac{z}{4}=1, \quad \Rightarrow \quad b=\frac{4}{3},\ \ c=4$$

Ox o‘qiga parallel tekislik.

3.3. Fazoda to‘g‘ri chiziq

1°.*Fazoda to‘g‘ri chiziqning vektor formadagi tenglamasi:*

$$\vec{r}=\overrightarrow{r_1}+t\cdot\vec{s} \qquad (1)$$



3.1-rasm

bu yerda $\vec{r}(x, y, z)$, $\overrightarrow{r_1}(x_1, y_1, z_1)$ – mos ravishda $M(x, y, z)\ va\ \ M_1(x_1, y_1, z_1)$ nuqtalarning radius vektorlari. $\vec{S}(m, n, p)$ – yo‘naltiruvchi vektor, t – esa parametr.

(1) tenglikdan *to‘g‘ri chiziqning parametr tenglamasi* kelib chiqadi:

$$\begin{cases} x = x_1 + mt \\ y = y_1 + nt \\ z = z_1 + pt \end{cases} \qquad (2)$$

(2) tenglikdan parametr t ni aniqlasak,

$$\frac{x-x_1}{m} = \frac{y-y_1}{n} = \frac{z-z_1}{p} \qquad (3)$$

to‘g‘ri chiziqning kanonik tenglamasini hosil qilamiz.

2°. Agar $M_1(x_1, y_1, z_1)$ va $M_2(x_2, y_2, z_2)$ nuqtalar berilgan bo‘lsa, *ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini*

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \qquad (4)$$

ko‘rinishda yozish mumkun, chunki $\vec{S} = \overrightarrow{M_1M_2} = (x_2 - x_{1,} y_2 - y_{1,} z_2 - z_1)$ vektor yo‘naltiruvchi vektor bo‘ladi.

3°. Ikkita to‘g‘ri chiziqning kanonik tenglamasi berilgan bo‘lsin:

$$\frac{x-x_1}{m_1} = \frac{y-y_1}{n_1} = \frac{z-z_1}{p_1}, \qquad \vec{S}_1(m_1, n_1, p_1)$$

$$\frac{x-x_2}{m_2} = \frac{y-y_2}{n_2} = \frac{z-z_2}{p_2}, \qquad \vec{S}_2(m_2, n_2, p_2)$$

ikki to‘g‘ri chiziqlar orasidagi burchak, ularning yo‘naltiruvchi vektor orasidagi burchak orqali ifodalanadi, shu sabab ikki vektorning skalyar ko‘paytmasidan foydalanib yozamiz:

$$cos\varphi = \frac{\vec{S}_1 \cdot \vec{S}_2}{|\vec{S}_1| \cdot |\vec{S}_2|} = \frac{m_1 m_2 + n_1 n_2 + p_1 p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2}\sqrt{m_2^2 + n_2^2 + p_2^2}} \tag{5}$$

Xususiy holda:

a) agar $\varphi = 0$ bo‘lsa $\vec{S}_1$va$\vec{S}_2$ vektor kollnear bo‘ladi, ya’ni

$\vec{S}_1 = \gamma \vec{S}_2$ tenglik o‘rinli bo‘ladi, bu erda

$$\gamma = \frac{\vec{S}_1}{\vec{S}_2}; \qquad \frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2} \tag{6}$$

to‘g‘ri chiziqlaring parallellik sharti kelib chiqadi

b) agar $\varphi = 90°$ bo‘lsa, $cos90° = 0$ bo‘lgani uchun (5) tenglikdan

$$m_1 m_2 + n_1 n_2 + p_1 p_2 = 0 \tag{7}$$

to‘g‘ri chiziqlaring perpendikulyarlik sharti kelib chiqadi.

4°. Fazoda to‘g‘ri chiziqning umumiy tenglamasi

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 & \vec{n}_1(A_1, B_1, C_1) \\ A_2 x + B_2 y + C_2 z + D_2 = 0, & \vec{n}_2(A_2, B_2, C_2) \end{cases} \tag{8}$$

ikkita tekislikning kesishishi natijasida to‘g‘ri chiziq hosil bo‘ladi. To‘g‘ri chiziqning yo‘naltiruvchi vektori tekisliklarning normal vektorlariga perpendikulyar bo‘ladi, shu sabab vektor ko‘paytmadan $\vec{S} = \vec{n}_1 \times \vec{n}_2$ yo‘naltiruvchi vektorning koordinatalarini aniqlash mumkin va to‘g‘ri chiziqdan ixtiyoriy nuqta olamiz:

$$\vec{S} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = m\vec{i} + n\vec{j} + p\vec{k}, \quad \vec{S}(m, n, p) \tag{9}$$

Tekisliklar sistemasini yechib $M_1(x_{1,}y_{1,}z_1)$ nuqta koordinatalari topiladi va to‘g‘ri chiziqning kanonik tenglamasi yoziladi

$$\frac{x - x_1}{m} = \frac{y - y_1}{n} = \frac{z - z_1}{p}$$

5°. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa

$$d = \frac{|\overrightarrow{M_1M_2} \cdot \vec{S}|}{|\vec{S}|} \qquad (10)$$

formula bilan aniqlanadi.

$M_2(x_{2,}y_{2,}z_2)$ nuqtadan $\frac{x-x_1}{m} = \frac{y-y_1}{n} = \frac{z-z_1}{p}$ to‘g‘ri chiziqqacha masofani topish uchun $\overrightarrow{M_1M_2} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$ va $\vec{S} = m\vec{i} + n\vec{j} + p\vec{k}$ vektorlaring vektor ko‘paytmasining modulini topamiz:

$$|\overrightarrow{M_1M_2} \cdot \vec{S}| = \sqrt{\begin{vmatrix} y_2 - y_1 & z_2 - z_1 \\ n & p \end{vmatrix} + \begin{vmatrix} x_2 - x_1 & z_2 - z_1 \\ m & p \end{vmatrix} + \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ m & n \end{vmatrix}} \qquad (11)$$

va vektor moduli $|\vec{S}| = \sqrt{m^2 + n^2 + p^2}$ ni topamiz va (10) formulaga qo‘ysak nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa kelib chiqadi.

3-misol. Piramida uchining koordinatalari berilgan:

$$A_1(3; 2; -2),\ A_2(6; -1; 7),\ A_3(1; 3; 5),\ A_4(-3; 0; 4)$$

1) A_1A_2va A_1A_4 qirralarining tenglamasini tuzing;

2) A_1A_2va A_1A_4 qirralar orasidagi burchakni toping;

3) A_3nuqtadan A_1A_4 to‘g‘ri chiziqgacha bo‘lgan masofani toping;

4) To‘g‘ri chiziqning kanonik tenglamasini tuzing;

5) Piramidaning $A_1A_2A_4$ yon yog‘ining yuzini toping;

6) Piramidaning hajmini toping;

1) A_1A_2va A_1A_4 qirralarining tenglamasini tuzish uchun (4) formuladan foydalanamiz:

$$A_1A_2: \qquad \frac{x-3}{6-3} = \frac{y-2}{-1-2} = \frac{z+2}{7+2} \Rightarrow \frac{x-3}{-3} = \frac{y-2}{-3} = \frac{z+2}{9}$$

A_1A_2 to‘g‘ri chiziq tenglamasi bo‘ladi, xuddi shuningdek A_1A_4 qirra tenglamasini tuzamiz:

$$A_1A_4: \qquad \frac{x-3}{-3-3}=\frac{y-2}{0-2}=\frac{z+2}{4+2} \Rightarrow \frac{x-3}{-6}=\frac{y-2}{-2}=\frac{z+2}{6}$$

bu A_1A_4 to‘g‘ri chiziq tenglamasi bo‘ladi.

2)A_1A_2va A_1A_4 qirralar orasidagi burchakni topamiz, bu to‘g‘ri chiziqlar orasidagi burchakshu to‘g‘ri chiziqlarning yo‘naltiruvchi vektorlari orasidagi burchak bilan mos bo‘ladi, shu sabab ikki vektor orasidagi burchakni (5) formuladan foydalanib topiladi:

A_1A_2 to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{S}_1(3;-3;9)$;

A_1A_4 to‘g‘ri chiziqning yo‘naltiruvchi vektori esa $\vec{S}_2(-6;-2;6)$ ga teng.

(5) formulaga asosan:

$$cos\varphi=\frac{\vec{S}_1\cdot\vec{S}_2}{|\vec{S}_1|\cdot|\vec{S}_2|}=\frac{3\cdot(-5)+(-3)\cdot(-2)+9\cdot 6}{\sqrt{3^2+(-3)^2+9^2}\sqrt{(-6)^2+(-2)^2+6^2}}=$$

$$=\frac{-18+6+54}{\sqrt{9+9+81}\sqrt{36+4+36}}=\frac{42}{\sqrt{99}\cdot\sqrt{76}}=\frac{42}{3\sqrt{11}\cdot 2\sqrt{19}}$$

$$=\frac{7}{\sqrt{209}}$$

$$\varphi=arccos\frac{7}{\sqrt{209}}$$

3)$A_3(1;3;5)$ nuqtadan A_1A_4 to‘g‘ri chiziqqacha bo‘lgan masofani hisoblaymiz. Buning uchun (10) formuladan foydalanamiz. (10) formulaning suratidagi ifoda $\overrightarrow{M_1M_2}\times\vec{S}$ vektor ko‘paytmaning modulidan iborat, bu qiymat (11) tenglik yordamida hisoblanadi:

$$\overrightarrow{M_1M_2} = \overrightarrow{A_1A_3} = (1-3;\ 3-2;\ 5+2) = (-2;1;7) \quad \Rightarrow \vec{S} = (-6;-2;6)$$

$$\left|\overrightarrow{A_1A_3} \times \vec{S}\right| = \sqrt{\begin{vmatrix} 1 & 7 \\ -2 & 6 \end{vmatrix}^2 + \begin{vmatrix} -2 & 7 \\ -6 & 6 \end{vmatrix}^2 + \begin{vmatrix} 1 & 7 \\ -2 & 6 \end{vmatrix}^2}$$

$$= \sqrt{(6+19)^2 + (-12+92)^2 + (9+6)^2}$$

$$= \sqrt{20^2 + 90^2 + 10^2} = 10\sqrt{4+9+1} = 10\sqrt{14}$$

Yo‘naltiruvchi vektor uzunligini topamiz:

$$\left|\vec{S}\right| = \sqrt{(-6)^2 + (-2)^2 + 6^2} = \sqrt{36+4+36} = \sqrt{76} = 2\sqrt{19}$$

Bu ko‘paytmalarni (10) formulaga keltirib berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofani topamiz:

$$d = \frac{\left|\overrightarrow{M_1M_2} \cdot \vec{S}\right|}{\overrightarrow{|S|}} = \frac{10\sqrt{14}}{\sqrt{19}} = 10\sqrt{\frac{14}{19}} \approx 10 \cdot 0{,}86 \approx 8{,}6$$

4) To‘g‘ri chiziqning umumiy tenglamasini kanonik ko‘rinishga keltirish uchun

$$\begin{cases} 10x + 13y + z - 54 = 0 \\ 3y + z - 4 = 0 \end{cases} \qquad \vec{n}_1 = (10;13;1), \quad \vec{n}_2 = (0;3;1)$$

$$\begin{cases} 10x + 13y + z - 54 \\ 3y + z = 4 \end{cases} => \begin{cases} 10x + 13y = 54 - z \\ 3y = 4 - 7 \end{cases}$$

o‘zgaruvchi zga ixtiyoriy qiymat beramiz, masalan, $z = 4$ qiymat bersak, sistema $\begin{cases} 10x + 13y = 50 \\ 3y = 0 \end{cases}$ ko‘rinishiga keladi, bu yerdan $y = 0$ va $x = 5$ ekanligi ma’lum bo‘ladi. $M(5;0;4)$ nuqta to‘g‘ri chiziqda yotuvchi ixtiyoriy nuqta bo‘ladi yo‘naltiruvchi vektorni $\vec{n}_1$ va $\vec{n}_2$ vektorlarning vektor ko‘paytmasidan topamiz, ya’ni

$$\vec{S} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 13 & 1 \\ 0 & 3 & 1 \end{vmatrix} = (13-3)\vec{i} - (10-0)\vec{j} + (30-0)\vec{k}$$

$$\vec{S} = 10\vec{i} - 10\vec{j} + 30\vec{k}$$

To‘g‘ri chiziqning kanonik tenglamasini yozamiz:

$$\frac{x-5}{10} = \frac{y}{-10} = \frac{z-4}{30}$$

5) Piramidaning $A_1A_2A_4$ yon yog‘ining yuzini hisoblash uchun A_1A_2 va A_1A_4 qirra uzunliklarini topamiz, buning uchun ikki nuqta orasidagi masofa $A_1A_2 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$ formulasidan foydalanamiz:

$$A_1A_2 = \sqrt{(6-3)^2 + (-1-2)^2 + (7+2)^2} = \sqrt{3^2 + (-3)^2 + 9^2}$$
$$= \sqrt{9+9+81} = \sqrt{99}$$

$$A_1A_4 = \sqrt{(-3-3)^2 + (0-2)^2 + (4+2)^2} = \sqrt{(-6)^2 + (-2)^2 + 6^2}$$
$$= \sqrt{36+4+36} = \sqrt{79}$$

Endi ular orasidagi burchak siniusini topamiz. Bizga $cos\varphi = \frac{7}{\sqrt{209}}$ ma’lum bo‘lgani uchun

$$sin\varphi = \sqrt{1 - cos^2\alpha} = \sqrt{1 - (\frac{7}{\sqrt{209}})^2} = \sqrt{1 - \frac{49}{209}} = \sqrt{\frac{160}{209}} = \frac{4\sqrt{10}}{\sqrt{209}}$$

Bizga ma’lum bo‘lgan formuladan foydalanib uchuburchak yuzini hisoblaymiz:

$S = \frac{1}{2} \cdot A_1A_2 \cdot A_1A_4 \cdot sin\varphi = \frac{1}{2} \cdot 3\sqrt{11} \cdot 2\sqrt{19} \cdot \frac{4\sqrt{10}}{\sqrt{19 \cdot 11}} = 12\sqrt{10}$ kv bir.

Piramidaning $A_1A_2A_4$ yon yog‘inining yuzi hisoblandi.

6) Piramidaning hajmini hisoblash uchun bizga ma’lum bo‘lgan formuladan foydalanamiz:

$$V = \frac{1}{6} \cdot S \cdot h = \frac{1}{6} \cdot 12\sqrt{10} \cdot \sqrt{10} = \frac{1}{6} \cdot 12 \cdot 10 = 20 \text{ kub birlik.}$$

3.4. To‘g‘ri chiziq bilan tekislik

1°. To‘g‘ri chiziqning kanonik tenglamasi va tekislikning umumiy tenglamasi berilgan bo‘lsin:

$$\frac{x - x_1}{m} = \frac{y - y_1}{n} = \frac{z - z_1}{p} \quad , \qquad \vec{S} = (m, n, p)$$

$$Ax + By + Cz + D = 0 \, , \qquad \vec{n} = (A, B, C)$$

ular *orasidagi burchakni topamiz*:

$$cos\alpha = \cos(90° - \varphi) = sin\varphi = \frac{\vec{n} \cdot \vec{S}}{|\vec{n}| \cdot |\vec{S}|} = \frac{Am + Bn + Cp}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{m^2 + n^2 + p^2}} \tag{12}$$

Xususiy holda:

a) $\varphi = 0°$ bo‘lsa

$$Am + Bn + Cp = 0 \tag{13}$$

to‘g‘ri chiziq va tekislik o‘zaro parallel bo‘ladi:

b) $\varphi = 90°$ bo‘lsa

$$\frac{A}{m} = \frac{B}{n} = \frac{C}{p} \tag{14}$$

to‘g‘ri chiziq va tekislik o‘zaro perendikulyar bo‘ladi.

2°. To‘g‘ri chiziq bilan tekislikning kesishish nuqtasini topish uchun to‘g‘ri chiziqning parametrik tenglamasidan foydalanamiz.

$$\begin{cases} x = x_1 + mt \\ y = y_1 + nt \\ z = z_1 + pt \end{cases} \tag{15}$$

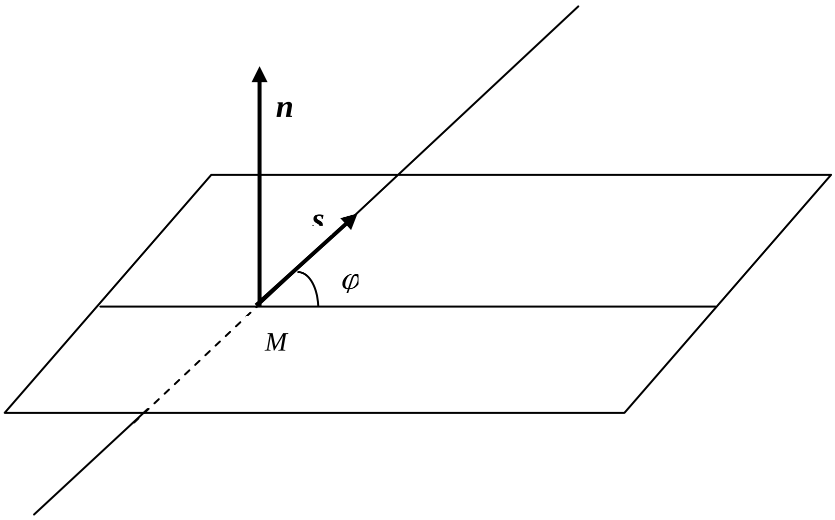
bularni tekislikning umumiy tenglamasiga qo‘yib parametr t ni topamiz:

$$A(x_1 + mt) + B(y_1 + nt) + C(z_1 + pt) + D = 0$$

bu erdan

$$t = -\frac{Ax_1 + By_1 + Cz_1 + D}{Am + Bn + Cp} \qquad (16)$$

ga teng bo‘ladi. (16) ni (15) ga qo‘yib kesishish nuqtasining koordinatalari topiladi(4.2-rasm).



3.2-rasm.

3°. Ikkita to‘g‘ri chiziqlarning bir tekislikda yotish shartini aniqlaymiz:

Quyidagi to‘g‘ri chiziqlar berilgan bo‘lsin:

$$\frac{x - x_1}{m_1} = \frac{y - y_1}{n_1} = \frac{z - z_1}{p_1} \vec{S}_1 = (m_1; n_1; p_1)$$

$$\frac{x - x_2}{m_2} = \frac{y - y_2}{n_2} = \frac{z - z_2}{p_2} \vec{S}_2 = (m_2; n_2; p_2)$$

va $\overrightarrow{M_1M_2} = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$ vektorlarga egamiz, bu vektorlar bir tekislikda yotsa, ya’ni ularning vektor-skalyar (aralash) ko‘paytmasi nolga teng bo‘lishi kerak

$$\overrightarrow{M_1M_2} \cdot \vec{S}_1 \cdot \vec{S}_2 = 0 \text{ ya'ni } \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0 \qquad (17)$$

Bu tenglik ikki to‘g‘ri chiziqning bir tekislikda yotish sharti bo‘ladi.

To‘g‘ri chiziq va tekislikka doir misol

4-misol. Piramida uchining koordinatalari berilgan:

$$A_1(3;2;-2),\quad A_2(6;-1;7),\quad A_3(1;3;5),\quad A_4(-3;0;4)$$

1) A_1A_3 to‘g‘ri chiziq bilan $A_1A_2A_4$ tekislik orasidagi burchak topilsin;

2) A_3 o‘tuvchi va $A_1A_2A_3$ tekislikka perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin;

3) $\frac{x+3}{10}=\frac{y}{13}=\frac{z-4}{1}$ to‘g‘ri chiziq bilan $10x+13y+z-54=0$ tekislikning kesishish nuqtasi topilsin;

Yechish:

1)A_1A_3 to‘g‘ri chiziq bilan $A_1A_2A_4$ tekislik orasidagi burchak topish uchun $A_1A_3$to‘g‘ri chiziqning kanonik tenglamasini tuzamiz:

$$\frac{x-3}{1-3}=\frac{y-2}{3-2}=\frac{z+2}{5+2}\Rightarrow\frac{x-3}{-2}=\frac{y-2}{1}=\frac{z+2}{7}\vec{S}(-2;1;7)$$

$A_1A_2A_4$ tekislik tenglamasi esa: $3y+z-4=0,\ \overrightarrow{n_1}(0;3;1)$ – normal vektori

$$sin\varphi=\frac{0\cdot(-2)+3\cdot1+1\cdot7}{\sqrt{0^2+3^2+1^2}\cdot\sqrt{(-2)^2+1^2+7^2}}=\frac{10}{\sqrt{10}\cdot\sqrt{54}}=\frac{\sqrt{10}}{3\sqrt{6}}=\frac{1}{3}\sqrt{\frac{5}{3}}$$

$$\varphi=arcsin\frac{1}{3}\sqrt{\frac{5}{3}}$$

2) $A_3(-1;0;4)$ o‘tuvchi va $A_1A_2A_3$ tekislikka perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi tuziladi: $\frac{A}{m}=\frac{B}{n}=\frac{C}{p}$

to‘g‘ri chiziq va tekislikning perpendikulyarlik shartidan topamiz:

$$A=10,\ B=13,\ C=1$$

$$\frac{m}{10}=\frac{n}{13}=\frac{p}{1}=A,\qquad m=\gamma10,\ n=\gamma13,\ p=\gamma$$

$$\frac{x+3}{10\gamma}=\frac{y-0}{13\gamma}=\frac{z-4}{\gamma}\Rightarrow\frac{x+3}{10}=\frac{y}{13}=\frac{z-4}{1}$$

3)$\frac{x+3}{10}=\frac{y}{13}=\frac{z-4}{1}$ to‘g‘ri chiziq bilan $10x+13y+z-54=0$ tekislikning kesishish nuqtasi topish uchun to‘g‘ri chiziqning kanonik tenglamasini parametrik ko‘rinishga keltiramiz:

$$\frac{x+3}{10}=\frac{y}{13}=\frac{z-4}{1}=t \quad \Rightarrow \quad \begin{cases} x=-3+10t \\ y=13t \\ z=4+t \end{cases}$$

Bu koordinatalarni $10x+13y+z-54=0$ tekislik tenglamasiga qo‘yamiz: $(-3+10t)\cdot 10+13\cdot 13t+4+t-54=0$

$$100t+159t+t-30-54+4=0$$

$270t=80 \quad \Rightarrow \quad t=\frac{8}{27}$, bundan t ning qiymatini parametrik tenglamalarga qo‘yib, topamiz: $x=-3+10\cdot\frac{8}{27}=-\frac{1}{27}$,

$$y=13\cdot\frac{8}{27}=\frac{104}{27},$$

$$z=4+\frac{8}{27}=\frac{116}{27}$$

Demak, to‘g‘ri chiziq bilan tekislik kesishish nuqtasi $M(-\frac{1}{27};\frac{104}{27};\frac{116)}{27}$ ga teng.

TOPSHIRIQ VARIANLARI

Hisob topshiriqlarini qabul qilishda beriladigan nazariy savollar

1. Tekislikda to‘g`ri chiziq tenglamalari.
2. Ikki to‘g`ri chiziq orasidagi burchak, ularning parallellik va perpendikulyarlik shartlari.
3. Tekislikning turli xil tenglamalari.
4. Ikki tekislik orasidagi burchak, ularning parallellik va perpendikulyarlik shartlari.
5. Fazoda to‘g`ri chiziq tenglamalari.
6. To‘g`ri chiziq va tekislik orasidagi burchak, ularning parallellik va perpendikulyarlik shartlari.

To‘g‘ri chiziq va tekislik bo‘limidan topshiriq variantlari.

1-TOPSHIRIQ.

A,B va C nutalar uchburchak uchlari bo‘lsa, quyidagilarni aniqlang:

a) AB tomoni tenglamasi;

b) CN balandlik tenglamasi;

c) AM mediana tenglamasi;

d) CN balandlik va AM medianalar kesishish nuqtasi;

e) C nuqtadan o‘tubchi va AB tomonga parallel to‘g‘ri chiziq tenglamasi;

f) C nuqtadan AB tomongacha bo‘gan masofa.

1. A(3;4), B(1;2), C(-2;-3)
2. A(-7;-5), B(-2;5), C(3;-2)
3. A(1;3), B(-1;4), C(-2;-3)
4. A(2;4), B(-3;-2), C(3;5)

5. A(-5;-4), B(1;4), C(3;2)
6. A(3;4), B(-2;3), C(4;-3)
7. A(-4;6), B(3;-5), C(2;6)
8. A(7;5), B(-4;-5), C(2;-3)
9. A(3;-2), B(-6;-2), C(1;1)
10. A(-5;-4), B(7;3), C(6;-2)
11. A(3;-5), B(-4;2), C(1;5)
12. A(7;4), B(1;-2), C(-5;-3)
13. A(-4;-7), B(-4;-5), C(2;-3)
14. A(-4;-5), B(3;1), C(5;7)
15. A(5;2), B(-3;5), C(1;-5)
16. A(-6;4), B(5;-7), C(4;2)
17. A(5;3), B(-3;-4), C(5;-6)
18. A(5;-4), B(-4;-6), C(3;2)
19. A(-7;-6), B(5;1), C(8;-4)
20. A(7;-1), B(1;7), C(3;7)
21. A(5;2), B(7;-6), C(-7;-6)
22. A(-2;-5), B(-6;-7), C(4;-5)
23. A(-6;-3), B(5;1), C(3;5)
24. A(7;4), B(-5;3), C(1;-5)
25. A(-8;2), B(3;-5), C(2;4)
26. A(4;3), B(2;7), C(-4;-2)
27. A(-9;-7), B(-4;3), C(5;-4)
28. A(3;5), B(-3;2), C(-3;-2)
29. A(4;2), B(-5;-4), C(5;7)
30. A(-4;-2), B(2;5), C(6;3)

2 -TOPSHIRIQ.

$A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3)$ va $A_4(x_4, y_4)$ nuqtalar berilgan.

Quyidagi tenglamalar tuzilsin:

a) $A_1A_2A_3$ tekislik;

b) A_1A_2 to‘g‘ri chiziq;

c) $A_1A_2A_3$ tekislikka perpendikulyar bo‘lgan A_4M to‘g‘ri chiziq;

d) A_1A_2 to‘g‘ri chiziqqa parallel A_3N to‘g‘ri chiziq;

e) A_4 nuqtadan o‘tib, A_1A_2 to‘g‘ri chiziqqa perpendikulyar bo‘lgan tekislik;

hisoblansin:

f) A_1A_4 to‘g‘ri chiziq bilan $A_1A_2A_3$ tekislik orasidagi burchak sinusi;

g) Oxy koordinata tekisligi bilan $A_1A_2A_3$ tekislik orasidagi burchakning kosinusi.

1. $A_1(3, 1, 4)$, $A_2(-1, 6, 1)$, $A_3(-1, 1, 6)A_4(0, 4, -1)$.

2. $A_1(3, -1, 2)$, $A_2(-1, 0, 1)$, $A_3(1, 7, 3)$, $A_4(8, 5, 8)$.

3. $A_1(3, 5, 4)$, $A_2(5, 8, 3)$, $A_3(1, 2, -2)$, $A_4(-1, 0, 2)$.

4. $A_1(2, 4, 3)$, $A_2(1, 1, 5)$, $A_3(4, 9, 3)$, $A_4(3, 6, 7)$.

5. $A_1(9, 5, 5)$, $A_2(-3, 7, 1)$, $A_3(5, 7, 8)$, $A_4(6, 9, 2)$

6. $A_1(0, 7, 1)$, $A_2(2, -1, 5)$, $A_3(1, 6, 3)$, $A_4(3, -9, 8)$.

7. $A_1(5, \ 5, 4)$, $A_2(1, -1, 4)$, $A_3(3, 5, 1)$, $A_4(5, 8, -1)$.

8. $A_1(6, 1, 1)$, $A_2(4, 6, 6)$, $A_3(4, 2, 0)$, $A_4(1, 2, 6)$.

9. $A_1(7, 5, 3)$, $A_2(9, 4, 4), A_3(4, 5, 7), A_4(7, 9, 6)$.

10. $A_1(6, 8, 2)$, $A_2(5, 4, 7)A_3(2, 4, 7)A_4(7, 3, 7)$.

11. $A_1(4, 2, 5)$, $A_2(0, 7, 1)$, $A_3(0, 2, 7)$, $A_4(1, 5, 0)$.

12. $A_1(4, 4, 10)$, $A_2(7, 10, 2)$, $A_3(2, 8, 4)$, $A_4(9, 6, 9)$.

13. $A_1(4, 6, 5)$, $A_2(6, 9, 4)$, $A_3(2, 10, 10)$, $A_4(7, 5, 9)$.

14. $A_1(3,5,4)$, $A_2(8,7,4), A_3(5,10,4), A_4(4,7,8)$.

15. $A_1(10,9,6)$, $A_2(2,8,2)$, $A_3(9,8,9)$, $A_4(7,10,3)$.

16. $A_1(1,8,2)$, $A_2(5,2,6)$, $A_3(5,7,4)$, $A_4(4,10,9)$.

17. $A_1(6,6,5)$, $A_2(4,9,5)$, $A_3(4,6,11)$, $A_4(6,9,3)$.

18. $A_1(7,2,2)$, $A_2(-5,7,-7)$, $A_3(5,-3,1)$, $A_4(2,3,7)$.

19. $A_1(8,-6,4)$, $A_2(10,5,-5)$, $A_3(5,6,-8)$, $A_4(8,10,7)$.

20. $A_1(1,-1,3)$, $A_2(6,5,8)$, $A_3(3,5,8)$, $A_4(8,4,1)$.

21. $A_1(1,-2,7)$, $A_2(4,2,10)$, $A_3(2,3,5)$, $A_4(5,3,7)$.

22. $A_1(4,2,10)$, $A_2(1,2,0)$, $A_3(3,5,7)$, $A_4(2,-3,5)$.

23. $A_1(2,3,5)$, $A_2(5,3,-7)$, $A_3(1,2,7)$, $A_4(4,2,0)$.

24. $A_1(5,3,7)$, $A_2(-2,3,5)$, $A_3(4,2,10)$, $A_4(1,2,7)$.

25. $A_1(4,3,5)$, $A_2(1,9,7)$, $A_3(0,2,0)$, $A_4(5,3,10)$.

26. $A_1(3,2,5)$, $A_2(4,0,6)$, $A_3(2,6,5)$, $A_4(6,4,-1)$.

27. $A_1(2,1,6)$, $A_2(1,4,9)$, $A_3(2,-5,8)$, $A_4(5,4,2)$.

28. $A_1(2,1,7)$, $A_2(3,3,6)$, $A_3(2,-3,9)$, $A_4(1,2,5)$.

29. $A_1(2,-1,7)$, $A_2(6,3,1)$, $A_3(3,2,8)$, $A_4(2,-3,7)$.

30. $A_1(0,4,5)$, $A_2(3,-2,1)$, $A_3(4,5,6)$, $A_4(3,3,2)$.

3-TOPSHIRIQ.

Quyidagi masalalar yechilsin.

1. Berilgan $M(-2,7,3)$ nuqtadan o‘tib, $x-4y+5z-1=0$ tekislikka parallel bo‘lgan tekislikning koordinata o‘qlaridan kesgan kesmalarining qiymatlari aniqlansin $\left(\text{Javob: } -\frac{1}{15}, \frac{4}{15}, -\frac{1}{3}\right)$.

2. Uchlari $\mathrm{M_1}(1,5,6)$ va $\mathrm{M_2}(-1,7,10)$ nuqtalarda bo‘lgan $\mathrm{M_1M_2}$ kesmaning o‘rtasidan o‘tib shu kesmaga perpendikulyar bo‘lgan tekislik tenglamasi tuzilsin (Javob: $\mathrm{x-y-2z+22=0}$).

3. $\mathrm{M}(2,0,-0{,}5)$ nuqtadan $4\mathrm{x}-4\mathrm{y}+2\mathrm{z}+17=0$ tekislikkacha bo‘lgan masofa topilsin (Javob: $\mathrm{d}=4$).

4. $\mathrm{A}(2,-3{,}5)$ nuqtadan o‘tib, Oxy tekislikka paralel bo‘lgan tekislik tenglamasi tuzilsin (Javob: $\mathrm{z}-5=0$).

5. $\mathrm{A}(2{,}5,-1)$ nuqta va $0\mathrm{x}$ o‘qi orqali o‘tuvchi tekislik tenglamasi tuzilsin (Javob: $\mathrm{y}+5\mathrm{z}=0$).

6. $\mathrm{A}(2{,}5,-1)$ va $\mathrm{B}(-3{,}1{,}3)$ nuqtalardan o‘tuvchi hamda $0\mathrm{y}$ o‘qiga parallel bo‘lgan tekislik tenglamasi tuzilsin (Javob: $4\mathrm{x}+5\mathrm{z}-3=0$).

7. $\mathrm{A}(3{,}4{,}0)$ nuqta va $\frac{\mathrm{x}-2}{1}=\frac{\mathrm{y}-3}{2}=\frac{z+1}{2}$ to‘g‘ri chiziq orqali o‘tuvchi tekislik tenglamasi tuzilsin (Javob: $\mathrm{y}-\mathrm{z}-4=0$).

8. O‘zaro parallel bo‘lgan $\frac{\mathrm{x}-3}{2}=\frac{\mathrm{y}}{1}=\frac{\mathrm{z}-1}{2}$ va $\frac{\mathrm{x}+1}{2}=\frac{\mathrm{y}-1}{1}=\frac{\mathrm{z}}{2}$ to‘g‘ri chiziqlar orqali o‘tadigan tekislik tenglamasi tuzilsin. (Javob: $\mathrm{x}+2\mathrm{y}-2\mathrm{z}-1=0$).

9. $\mathrm{A}(3{,}2,-5)$ nuqta bilan Oxo‘qi orqali o‘tuvchi tekislikning $3\mathrm{x}-\mathrm{y}-7\mathrm{z}+9=0$ tekislik bilan kesishishidan hosil bo‘lgan to‘g‘ri chiziqning umumiy tenglamasi tuzilsin.($Javob$: $3x-y-7z+9=0{,}5y+2z=0$).

10. Agar tekislik $M(6,-10,1)$ nuqtadan o‘tib, Ox va Oz o‘qlardan mos ravshida $a=-3$ va $c=2$ kesmalar kesadigan bo‘lsa, uning kesmalar bo‘yicha tenglamasi tuzilsin $\left(\text{Javob:} \frac{x}{-3}+\frac{y}{-4}+\frac{z}{2}=1\right)$

11. $A(2,3,-4)$ nuqtadan o‘tib, $\vec{a}=(4,1,-1)$va$\vec{b}=(2,-1,2)$ vektorlarga parallel bo‘lgan tekislik tenglamasi tuzilsin $(\text{Javob:} x-10y-6z+4=0)$.

12. $A(1,1,0)$ va $B(2,-1,-1)$ nuqtalardan o‘tuvchi hamda $5x+2y+3z-7=0$ tekislikka perpendikulyar bo‘lgan tekislik tenglamasi tuzilsin.

$(\text{Javob:} x+2y-3z-3=0)$.

13. Koordinata boshidan o‘tib, $2x-3y+z-1=0$ va$x-y+5z+3=0$ tekisliklarga perpendikulyar bo‘lgan tekislik tenglamasi tuzilsin

$(\text{Javob:} 14x+9y-z=0)$.

14. $A(3,-1,2)$va$B(2,1,4)$ nuqtalardan o‘tuvchi hamda $\vec{a}=(5,-2,-1)$ vektorga parallel bo‘lgan tekislikning tenglamasi tuzilsin.$(\text{Javob:} 2x+9y-8z++19=0)$.

15. Agar $A(5,-2,3)$va$B(1,-3,5)$ nuqtalar $\overrightarrow{AB}$ vektorning boshi va oxiri bo‘lsa, koordinata boshidan o‘tib, $\overrightarrow{AB}$ vektorga perpendikulyar bo‘lgan tekislik tenglamasi tuzilsin. $(\text{Javob:} 4x+y-2z=0)$

16. $M(2,-3,3)$ nuqtadan o‘tib, $3x+y-3z=0$ tekislikka parallel bo‘lgan tekislikning koordinata o‘qlaridan kesgan kesmalarning qiymatlari topilsin. $(\text{Javob:} -2,-6,2.)$

17. Uchlari $M_1(2,3,-4)$va$M_2(-1,2,-3)$ nuqtalarda bo‘lgan M_1M_2 kesmaga perpendikulyar bo‘lib, $M(1,-1,2)$ nuqtadan o‘tuvchi tekislik tenglamasi tuzilsin. (Javob: $3x+u-z=0$).

18. $\frac{x}{6}=\frac{y-3}{8}=\frac{z-1}{9}$ to‘g‘ri chiziqning $x+3u-2z-1=0$ tekislikka parallel ekanligi hamda $x=t+7, y=t-2,\ z=2t+1$ to‘g‘ri chiziqning ushbu tekislikda yotishi ko‘rsatilsin.

19. Koordinata tekisligi Oxz ga parallel bo‘lib, $A(3,-4,1)$ nuqtadan o‘tuvchi tekislikning umumiy tenglamasi yozilsin (Javob: $y+4=0$).

20. Ordinata o‘qi bo‘yicha hamda $M(3,-5,2)$ nuqtadan o‘tuvchi tekislik tenglamasi tuzilsin. (Javob: $2x-3z=0$).

21. $M(1,2,3)$va$N(-3,4,-5)$ nuqtalardan o‘tib Oz o‘qiga parallel bo‘lgan tekislik tenglamasi tuzilsin (Javob: $x+2y-5=0$)

22. $M(2,3,-1)$ nuqtadan hamda $x=t-3,\ y=2t+5,\ z=-3t+1$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasi tuzilsin (Javob: $10x+13y+12z-47=0$).

23. $M(4,-3,1)$ nuqtaning $x-2y-z-15=0$ tekislikdagi proyeksiyasi topilsin. (Javob: $M_1(5,-5,0)$).

24. $x-4y+z-1=0$ bilan $2x+bu+10z-3=0$ tekisliklar b ning qanday qiymatida o‘zaro perpendikulyar bo‘ladilar? (Javob: $b=3$).

25. Koordinata o‘qlaridan 0 dan farqli bir xil o‘lchamdagi kesmalar kesadigan hamda $M(2,-3,-4)$ nuqtadan o‘tadigan tekislik tenglamasi tuzilsin.

(Javob: $x+y+z+5=0$).

26. $\frac{x}{3}=\frac{y-5}{n}=\frac{z+5}{6}$ to‘g‘ri chiziq bilan $ax+2y-2z-7=0$ tekislik, n va a ning qanday qiymatlarida o‘zaro perpendikulyar bo‘ladi? (Javob: $a=-1,\ n=-6$).

27. Agar tekislik $A(2,3,-1)$va$B(1,1,4)$nuqtalardan o‘tib, $x-4u+3z+2=0$ tekislikka perpendikulyar bo‘lsa, uning tenglamasi tuzilsin. (Javob: $7x+4y+3z-23=0$).

28. Ikkita $x+5y-z+7=0$ va $3x-y+2z-3=0$ tekisliklarga perpendikulyar bo‘lgan va koordinata boshidan o‘tadigan tekislik tenglamasi tuzilsin (Javob: $9x-5y-16z=0$).

29. Ikkita $M(2,3,-5)$ va $N(-1,1,-6)$ nuqtalardan o‘tib, $\vec{a}=(4,4,3)$ vektorga parallel bo‘lgan tekislik tenglamasi tuzilsin.

30. Ikkita $3x-5y+cz-3=0$ bilan $x-3\ +2z+5=0$ tekisliklar ***c*** ning qanday qiymatiga o‘zaro perpendikulyar bo‘ladi?

4 -TOPSHIRIQ.

Quyidagi masalalar yechilsin.

1)$\frac{x-1}{6}=\frac{y+2}{2}=\frac{z}{-1}$ to‘g‘ri chiziq va $\begin{cases} x-2y+2z-8=0 \\ x+6z-6=0 \end{cases}$ to‘g‘ri chiziqlarning o‘zaro parallel ekanliklari isbotlansin.

2)$\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-3}{3}$ to‘g‘ri chiziq bilan $2x+y-z=0$ tekislikning o‘zaro parallel ekanligi hamda $\frac{x-2}{2}=\frac{y}{-1}=\frac{z-4}{3}$ to‘g‘ri chiziqning esa, ushbu tekislikda yotishi isbotlansin.

3)Koordinata o‘qlari bilan mos ravishda $60^0, 45^0$va120^0 li burchak tashkil etuvchi hamda $M(1,-3,3)$ nuqtadan o‘tuvchi to‘g‘ri chiziq, tenglamasi tuzilsin. (Javob: $\frac{x-1}{1}=\frac{y+3}{\sqrt{2}}=\frac{z-3}{-1}.$)

4)$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-1}{6}$ to‘g‘ri chiziqning $\begin{cases} 2x+y-4z+2=0 \\ 4x-y-5z+4=0 \end{cases}$ to‘g‘ri chiziqqa perpendikulyar ekanligi isbotlansin.

5)Uchlari $A(3,6,-7), B(-5,1,-4)$va $C(0,2,3)$nuqtalarda bo‘lgan uchburchakning S uchidan tushirilgan medianasining parametrik tenglamalari tuzilsin (Javob: $x=2t, y=-3t+2, z=17t+3.$)

6)Quyidagi $\frac{x+2}{3}=\frac{y-1}{n}=\frac{z}{1}$ to‘g‘ri chiziq, n ning qanday qiymatida $\begin{cases} x+y-z=0 \\ x-y-5z-8=0 \end{cases}$ to‘g‘ri chiziqqa parallel bo‘ladi? (Javob: $n=-2$).

7)$\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z}{6}$ to‘g‘ri chiziq va $2x+3y+z-1=0$ tekislikning kesishish nuqtasi topilsin (Javob: $M(2,-3,6)$).

8)$P(3,1,-1)$ nuqtaning $x+2y+3z-30=0$ tekislikdagi proyeksiyasi topilsin (Javob: $P_1(5,5,5)$).

9)Ikkita $3x-5y+cz-3=0$, $x+3y+2z+5=0$ tekislik *c*ning qanday qiymatida o‘zaro perpendikulyardir? (Javob: $c=6$).

10) $ax+3y-5z+1=0$ tekislik *a* ning qanday qiymatida $\frac{x-1}{4}=\frac{y+2}{3}=\frac{z}{1}$ to‘g‘ri chiziqqa parallel bo‘ladi? (Javob: $a=-1$).

11) $m$va$S$ning qanday qiymatlarida $\frac{x-2}{m}=\frac{y+1}{4}=\frac{z-5}{-3}$ to‘g‘ri chiziq bilan $3x-2y+cz+1=0$ tekislik o‘zaro perpendikulyar bo‘ladi? (Javob: $m=-6, \quad c=1{,}5$.

12) Koordinata boshidan o‘tuvchi va $x = 2t + 5,\ y = -3t + 1,$ $z = -7t - 4$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin (Javob: $\frac{x}{2} = \frac{y}{-3} = \frac{z}{-7}$).

13) $A(0,0,2),\ B(4,2,5),$ va $C(12,6,11)$ nuqtalarning bitta to‘g‘ri chiziqda yotishi yoki yotmasliklari tekshirilsin (Javob: yotadi).

14) Berilgan $M(2,-5,3)$ nuqtadan o‘tib $\begin{cases} 2x - y + 3z - 1 = 0 \\ 5x + 4y - z - 7 = 0 \end{cases}$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin.

15) Berilgan $M(2,-3,4)$ nuqtadan o‘tib $\frac{x+2}{1} = \frac{y-3}{-1} = \frac{z+1}{1}$ va $\frac{x+4}{2} = \frac{y}{1} = \frac{z-4}{-3}$ to‘g‘ri chiziqlarga perpendikulyar bo‘lgan to‘g‘richiziq tenglamasi tuzilsin

16) $ax + by + 6z - 7 = 0$ tekislik, a va b ning qanday qiymatlarida $\frac{x-2}{2} = \frac{y+5}{-4} = \frac{z+1}{-9}$ to‘g‘ri chiziqqa perpendikulyar bo‘ladi? (Javob: $a = 4,$ $b = -8$).

17) $\frac{x}{6} = \frac{y-3}{-8} = \frac{z-1}{-9}$ to‘g‘ri chiziqning $x + 3y - 2z + 1 = 0$ tekislikka parallel ekanligi va $x = t + 7, y = t - 2, z = 2t + 1$ to‘g‘ri chiziqning esa,ushbu tekislikda yotishi ko‘rsatilsin.

18) $k(-3,1,-2)$ nuqtadan hamda Oz o‘q orqali o‘tuvchi tekislik tenglamasi tuzilsin (Javob: $x + 3y = 0$).

19) $\frac{x}{1} = \frac{y-1}{-2} = \frac{z}{3},$ $\begin{cases} 3x + y - 5z + 1 = 0 \\ 2x + 3y - 8z + 3 = 0 \end{cases}$ to‘g‘ri chiziqlar o‘zaro perpendikulyar ekanligi ko‘rsatilsin.

20) d ning qanday qiymatida $\begin{cases} 3x - y + 2z - 6 = 0 \\ x + 4y - z + d = 0 \end{cases}$ to‘g‘ri chiziq Oz o‘qini kesadi? (Javob: $d = 3$).

21) Ushbu $\begin{cases} x = 2t + 5 \\ y = -t + 2 \\ z = pt - 7 \end{cases}$ va $\begin{cases} x + 3y + z + 2 = 0 \\ x - y - 3t - 2 = 0 \end{cases}$ to‘g‘ri chiziqlar p ning qanday qiymatida parallel bo‘ladi.(Javob: $p = -5$).

22) . $\frac{x-7}{5} = \frac{y-1}{1} = \frac{z-5}{4}$ to‘g‘ri chiziq bilan $3x - y + 2z - 8 = 0$ tekislikning kesishish nuqtasi topilsin. (Javob: $M(2,0,1)$).

23) Berilgan $A(2,-5,3)$ nuqtadan o‘tib 0xz tekislikka parallel bo‘lgan tekislikning tenglamasi tuzilsin. (Javob: $\mathrm{y}+5=0$).

24) Berilgan $M(5,3,2)$ nuqtadan va Oy o‘qi orqali o‘tadigan tekislik bilan $\mathrm{x}+2\mathrm{y}-\mathrm{z}+5=0$ tekislikda kesishishidan hosil bo‘lgan to‘g‘ri chiziqning umumiy tenglamasi tuzilsin.

25) b va d ning qanday qiymatlarida $\begin{cases} x-2y+z-9=0 \\ 3x+by+z+d==0 \end{cases}$ to‘g‘ri chiziq Oxy tekislikda yotadi? (Javob: $b=-6,\ d=-27$).

26) Berilgan $M_0(2,3,3)$ nuqtadan o‘tib, $\vec{a}=(-1,-3,1)\ va\ \vec{b}=(4,1,6)$ vektorlarga parallel bo‘lgan tekislik tenglamasi tuzilsin. (Javob: $19\mathrm{x}-10\mathrm{y}-11\mathrm{z}+25=0$).

27) Ox o‘qiga parallel bo‘lib $E(3,4,5)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi tuzilsin. $\left(\text{Javob: } \frac{\mathrm{x}-3}{1}=\frac{\mathrm{y}-4}{0}=\frac{\mathrm{z}-5}{0}\right)$.

28) Shunday bir to‘g‘ri chiziq tenglamasi tuzilsinki, u $\mathrm{M}(2,3,1)$ nuqtadan o‘tib, $\frac{\mathrm{x}+1}{2}=\frac{\mathrm{y}}{-1}=\frac{\mathrm{z}-2}{3}$ to‘g‘ri chiziqqa perpendikulyar bo‘lsin.

29) Berilgan $\mathrm{M}(1,-5,3)$ nuqtadan o‘tuvchi hamda $\frac{\mathrm{x}}{2}=\frac{\mathrm{y}-2}{3}=\frac{\mathrm{z}+1}{-1}$ va $\mathrm{x}=3\mathrm{t}+1,\ \mathrm{y}=-\mathrm{t}-5,\ \mathrm{z}=2\mathrm{t}+3$ to‘g‘ri chiziqlarga perpendikulyar bo‘lgan to‘g‘ri chiziqning kanonik tenglamasi yozilsin. (Javob: $\frac{\mathrm{x}-1}{5}=\frac{\mathrm{y}+5}{-7}=\frac{\mathrm{z}-3}{-11}$).

30) $\mathrm{M}(4,3,10)$ nuqtaga $\frac{\mathrm{x}-1}{2}=\frac{\mathrm{y}-2}{4}=\frac{\mathrm{z}-3}{5}$ to‘g‘ri chiziqqa nisbatan simmetrik nuqta topilsin (Javob: $\mathrm{M}_1(2,9,6)$).

IV. MATEMATIK ANALIZGA KIRISH

4.1 KETMA-KETLIKLAR VA FUNKSIYALARNING LIMITLARI.

Aytaylik, biror $\{x_n\}$ sonli ketma-ketlik qaralayotgan bo‘lsin. Ta’rifga binoan, agar har qanday ε>0 son uchun Shunday bir $N=N(\varepsilon)>0$ butun son mavjud bo‘lib, barcha $n>N$ lar uchun $|x_n - A| < E$ tengsizlik o‘rinli bo‘ladigan bo‘lsa, A soni $\{x_n\}$ *ketma-ketlikning limiti* deb atalib, uni $\lim\limits_{n\to\infty} x_n = A$ kabi yoziladi. Chekli limiti mavjud bo‘lgan ketma-ketlikni *yaqinlashuvchi,* aks holda *uzoqlashuvchi* deyiladi.

1-misol. $\{x_n\} = \left\{\frac{2n+3}{n+1}\right\}$ ketma-ketlikning limiti A=2 ekanligi ko‘rsatilsin.

Yechish: Har qanday $\varepsilon > 0$ son uchun Shunday $N=N(\varepsilon)>0$ son mavjud bo‘lib, barcha $n>N$ lar uchun $|x_n - A| = \left|\frac{2n+3}{n+1} - 2\right| = \left|\frac{2n+3-2n-2}{n+1}\right| = \frac{1}{n+1} < \varepsilon$ ning bajarilishini ko‘rsatamiz. $\frac{1}{n+1} < \varepsilon$ dan, $n > \frac{1}{\varepsilon} - 1$ ni hosil qilamiz. Demak, $N = \left[\frac{1}{\varepsilon} - 1\right] + 1$ (bu erda [α], α sonining butun qismidir). Bundan ko‘rinmoqdaki, Shunday N son mavjud ekanki, barcha $n>N$ lar uchun $|x_n - 2| < \varepsilon$ kabi shart bajariladi.

Faraz qilaylik, $y=f(x)$ funksiya biror x_0 nuqta atrofida aniqlangan bo‘lsin. U holda, agar har qanday ε>0 son uchun Shunday bir $\delta=\delta(E)>0$ son mavjud bo‘lib, barcha $0 < |x - x_0| < \delta$ lar uchun $|f(x) - A| < \varepsilon$ tengsizlik o‘rinli bo‘lsa, chekli A sonini $y=f(x)$ *funksiyaning* $x \to x_0$ *(*$x=x_0$ *nuqtada) dagi limiti* deb ataladi hamda $\lim\limits_{x\to x_0} f(x) = A$ yoziladi. Ayrim hollarda $x=x_0$ nuqtada funksiya aniqlanmagan ham bo‘lishi mumkin. Agar har qanday E>0 son uchun $N=W(E)>0$ mavjud bo‘lib, barcha $|x| > N$ lar uchun

$|f(x)-A|<E$ bajariladigan bo‘lsa, u holda $\lim_{x\to\pm\infty} f(x)=A$ deb yoziladi. Agar $\lim_{\substack{x\to x_0\\ x<x_0}} f(x)$ kabi (uni $\lim_{x\to x_0-0} f(x)$ yoki $f(x_0-0)$ deb ham yozish mumkin) chekli limit mavjud bo‘lsa, u limitni $f(x)$ funksiyaning x_0 nuqtadagi *chap bir tomonli limiti* deb ataladi. Shunga o‘xshash $\lim_{\substack{x\to x_0\\ x>x_0}} f(x)$ (yoki $\lim_{x\to x_0+0} f(x)$ yoki $f(x_0+0)$) chekli limit mavjud bo‘lsa, u holda uni $f(x)$ funksiyaning x_0 nuqtadagi *o‘ng bir tomonli limiti* deb ataladi. Biror $f(x)$ funksiyaning belgilangan x_0 nuqtadagi chekli limiti mavjud bo‘lishi uchun, uning shu nuqtadagi ham chap bir tomonli, ham o‘ng bir tomonli chekli limitlari mavjud bo‘lib, $f(x_0-0)=f(x_0+0)$ tenglikning bajarilishi zarur hamda yetarli shartdir.

Limitlar uchun quyidagi teoremalar o‘rinlidir:

1-teorema. Agar $\lim_{x\to x_0} f_i(x)$ $(i=\overline{1,n})$ chekli limit bo‘lsa, u holda:

$$\lim_{x\to x_0}\sum_{i=1}^{n} f_i(x)=\sum_{i=1}^{n}\lim_{x\to x_0} f_i(x), \qquad \lim_{x\to x_0}\prod_{i=1}^{n} f_i(x)=\prod_{i=1}^{n}\lim_{x\to x_0} f_i(x)$$ lar o‘rinlidir.

2-teorema. Agar $\lim_{x\to x_0} f(x)$ va $\lim_{x\to x_0}\varphi(x)\neq 0$ chekli limitlar mavjud bo‘lsa, u holda: $\lim_{x\to x_0}\frac{f(x)}{\varphi(x)}=\lim_{x\to x_0} f(x)/\lim_{x\to x_0}\varphi(x)$ tenglik o‘rinlidir.

Yuqoridagi teoremalar $x_0=\pm\infty$ uchun ham o‘z kuchida qolaveradi. Agar yuqoridagi teoremalarning shartlari bajarilmasa, u holda $\infty-\infty$, $\frac{\infty}{\infty},\frac{0}{0}$ va boshqa xildagi aniqmasliklar paydo bo‘ladiki, ularni sodda hollar uchun algebraik shakl almashtirishlar orqali aniq ko‘rinishga keltiriladi.

2-misol. $\lim_{x\to 2}\left(\frac{4}{x^2-4}-\frac{1}{x-2}\right)$ ni hisoblansin.

Yechish: Ushbu limitda $(\infty-\infty)$ turdagi aniqmaslik yuz bermoqda. Uni ochib chiqish maqsadida, umumiy maxrajga keltirilganda $\lim_{x\to 2}\frac{2-x}{x^2-4}$ dan $\frac{0}{0}$

aniqmaslikka kelamiz. Agar x-2≠0 ga qisqartirsak, $\lim_{x\to 2}\left(-\frac{1}{x+2}\right)=-\frac{1}{4}$ hosil bo‘ladi.

3-misol . $\lim_{x\to\pm\infty}\frac{2x^3-x+5}{x^3+x^2-1}$ ni hisoblansin.

Yechish: Bu yerda $\frac{\infty}{\infty}$ aniqmaslik ishtirok etmoqda. Kasrning surat va maxrajini x^3 ga bo‘lib yuborsak, $\lim_{x\to\pm\infty}\frac{2-\frac{1}{x^2}+\frac{5}{x^3}}{1+\frac{1}{x}-\frac{1}{x^3}}$ ni hosil qilamiz. Natijada, limitga o‘tib 2 ni hosil qilamiz.

AJOYIB LIMITLAR

Quyidagi 1) $\lim_{x\to 0}\frac{\sin x}{x}=1;$

$$2)\ \lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=\lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e\approx 2{,}71828;$$

limitlar ajoyib limitlar deb atalib, ular limitlarni hisoblashda keng qo‘llaniladi.

1-misol . $\lim_{n\to 0}\frac{\sin 7x}{\sin 3x}$ hisoblansin.

Yechish: Agar x≠0 deb olsak, u holda

$$\lim_{n\to 0}\frac{\sin 7x}{\sin 3x}=\lim_{n\to 0}\frac{\frac{\sin 7x}{7x}\cdot 7x}{\frac{\sin 3x}{3x}\cdot 3x}=\lim_{n\to 0}\left(\frac{7x}{3x}\right)\cdot\lim_{n\to 0}\frac{\frac{\sin 7x}{7x}}{\frac{\sin 3x}{3x}}=\frac{7}{3}.$$

2-misol . $\lim_{n\to\infty}\left(\frac{2x+1}{2x-1}\right)^{3x+1}$ ni hisoblansin.

Yechish: Quyidagicha shakl almashtiramiz:

$$\lim_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{3x+1}=\lim_{x\to\infty}\left(\frac{2x-1+2}{2x-1}\right)^{3x+1}=\lim_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{3x+1};$$

Agar $\frac{2}{2x-1}=\frac{1}{y}$ deb belgilash kiritsak, $x=y-\frac{1}{2}$ dan $x\to\infty$ da $u\to\infty$.

$$\lim_{x\to\infty}\left(\frac{2}{2x-1}\right)^{3x+1}=\lim_{x\to\infty}\left(1+\frac{1}{y}\right)^{3x-\frac{1}{2}}=\lim_{y\to\infty}\left(\left(1+\frac{1}{y}\right)^{y}\right)^{3}\cdot\left(1+\frac{1}{y}\right)^{-\frac{1}{2}}=e^{3}$$

ni hosil qilamiz.

4.2 CHEKSIZ KICHIK FUNKSIYALARNI TAQQOSLASH. FUNKSIYANING UZLUKSIZLIGI

Agar $\lim\limits_{x\to x_0}\alpha(x)=0$ bo‘lsa, u holda $\alpha(x)$ ni $x\to x_0$ dagi *cheksiz kichik miqdor* deb ataladi. Ikkita $\alpha(x)$ va $\beta(x)$ ($x\to x_0$ dagi) cheksiz kichik miqdorlarni taqqoslash uchun ularning nisbatining limiti hisoblanadi:

$$\lim_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=C \tag{4.1}$$

Agar $C\neq 0$ bo‘lsa, $\alpha(x)$ bilan $\beta(x)$ larni *bir xil tartibli cheksiz kichik miqdorlar* deb yuritiladi; agar $C=0$ bo‘lsa, $\alpha(x)$ni $\beta(x)$ ga nisbatan *yuqori tartibli cheksiz kichik miqdor* deb, $\beta(x)$ ni esa, $\alpha(x)$ ga nisbatan *quyi tartibli cheksiz kichik miqdorlar* deb ataladi.

Agar $\lim\limits_{x\to x_0}\frac{\alpha(x)}{(\beta(x))^k}=C$, $(0<|C|<\infty)$, bo‘lsa, $\alpha(x)$ ni $x\to x_0$ da $\beta(x)$ ga nisbatan k-tartibli cheksiz kichik miqdor deyiladi.

Agar $\lim\limits_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=1$ bo‘lsa, $\alpha(x)$ bilan $\beta(x)$ funktsilarni $x\to x_0$ da *ekvivalent (teng kuchli) cheksiz miqdorlar* deb atalib, ularni $\alpha(x)\sim\beta(x)$ deb yoziladi. Masalan, $x\to 0$ da $\sin ax\sim ax$, $tgx\sim x$, $\ln(1+x)\sim x$, $e^{ax}-1\sim ax$. Agar $x\to x_0$ da $\alpha(x)$ bilan $\beta(x)$ lar, hamda $\alpha^*(x)$ bilan $\beta^*(x)$ lar cheksiz kichik miqdorlar bo‘lib, $\alpha(x)\sim\alpha^*(x)$ va $\beta(x)\sim\beta^*(x)$ bo‘lsalar, u holda:

$$\lim_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=\lim_{x\to x_0}\frac{\alpha^*(x)}{\beta(x)}=\lim_{x\to x_0}\frac{\alpha(x)}{\beta^*(x)}=\lim_{x\to x_0}\frac{\alpha^*(x)}{\beta^*(x)}$$

ekanligini isbotlash mumkin.

1-misol . $\lim\limits_{x\to x_0}\frac{\sin 5x}{\ln(1+x)}$ ni hisoblansin.

Yechish: Agar $x\to 0$ da $\sin 5x \sim 5x$ va $\ln(1+x)\sim x$ ekanligini inobatga olsak, $\lim\limits_{x\to x_0}\frac{\sin 5x}{\ln(1+x)}=\frac{5x}{x}=5$ hosil bo‘ladi.

Agar $u=f(x)$ funksiya biror x nuqtada va uning atrofida aniqlangan bo‘lib, x nuqtada uning chekli limiti mavjud bo‘lsa hamda u limit funksiyaning x_0 nuqtadagi qiymatiga teng bo‘ladigan bo‘lsa, yaэni:

$$\lim_{x\to x_0} f(x)=f(x_0) \tag{4.2}$$

u holda $u=f(x)$ funksiyani $x=x_0$ *nuqtada uzluksiz* deb ataladi.

Agar $x=x_0+\Delta x$ olinsa, (5.2) uzluksizlik sharti quyidagi $\lim\limits_{\Delta x\to 0}\Delta f(x_0)=\lim\limits_{\Delta x\to 0}\left(f(x_0+\Delta x)-f(x_0)\right)=0$ shartga teng kuchlidir, yaэni, x_0 nuqtadagi argumentning cheksiz kichik orttirmasi Δx ga, funksiyaning ham $\Delta f(x_0)$ cheksiz kichik orttirmasi mos keladigan bo‘lsagina, $u=f(x)$ funksiya $x=x_0$ nuqtada uzluksiz bo‘ladi. Agar $u=f(x)$ funksiya biror sohaning barcha nuqtalarida uzluksiz bo‘lsa, uni *shu sohada uzluksiz* deb ataladi.

2-misol . Har qanday $x\in R$ uchun $u=\sin 5x$ funksiyaning uzluksizligi isbotlansin.

Yechish: Argumentning ixtiyoriy qiymatidagi Δx orttirma uchun funksiyaning orttirmasi

$$\Delta y=\sin(5x+\Delta x)-\sin 5x=2Cos\left(5x+\frac{5}{2}\Delta x\right)\cdot\sin\frac{5}{2}\Delta x$$ mos keladi.

U holda: $$\lim_{\Delta x\to 0}\Delta y=2\lim_{\Delta x\to 0}Cos\left(5x+\frac{5}{2}\Delta x\right)\lim_{\Delta x\to 0}\sin\frac{5}{2}\Delta x=0$$

Demak, $u=\sin 5x$ funksiya son o‘qining barcha qiymatlarida uzluksiz ekan.

Agar x_0 nuqtadagi uzluksizlik shartlaridan hech bo‘lmaganda bittasi bajarilmasa, x_0 ni *funksiyaning uzilish nuqtasi* deb ataladi. Xususan, agar $u=f(x)$ funksiyaning x_0 nuqtadagi chekli bir tomonli limitlari $f(x_0-0)$ va $f(x_0+0)$ mavjud bo‘lib, $f(x_0-0)\neq f(x_0+0)$ bo‘lsa, x_0 ni *1-tur uzilish nuqtasi* deyiladi. Agar x_0 ni nuqtadagi bir tomonli $f(x_0-0)$, $f(x_0+0)$ limitlardan hech bo‘lmaganda birortasi mavjud bo‘lmasa yoki cheksizlikka teng bo‘lsa, x_0 ni *2-tur uzilish nuqtasi* deb ataladi. Agar $f(x_0-0)=f(x_0+0)$ bo‘lib, $f(x)$ funksiya x_0 nuqtada aniqlanmagan bo‘lsa, u holda x_0 ni funksiyaning *chetlantiriladigan uzilish nuqtasi* deyiladi. Masalan, $y=\frac{\sin x}{x}$ uchun, $x=0$ chetlantiriladigan uzilish nuqtasidir.

LIMITLAR NAZARIYASI BO‘LIMIGA TOPSHIRIQ VARIANTLARI

Ko‘rsatilgan limitlar hisoblansin.

1 – TOPSHIRIQ.

1.1. $\lim\limits_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 12x + 20}$.

1.2. $\lim\limits_{x \to 0} \dfrac{x^3 - x^2 + 2x}{x^2 + x}$.

1.3. $\lim\limits_{x \to 3} \dfrac{6 + x - x^2}{x^3 - 27}$.

1.4. $\lim\limits_{x \to 1} \dfrac{2x^2 - x - 1}{3x^2 - x - 2}$.

1.5. $\lim\limits_{x \to 2} \dfrac{2x^2 - 7x + 4}{x^2 - 5x + 6}$.

1.6. $\lim\limits_{x \to 3} \dfrac{12 + x - x^2}{x^3 - 27}$.

1.7. $\lim\limits_{x \to 1/3} \dfrac{3x^2 + 2x - 1}{27x^3 - 1}$.

1.8. $\lim\limits_{x \to -1} \dfrac{x^2 - 4x - 5}{x^2 - 2x - 3}$.

1.9. $\lim\limits_{x \to -1} \dfrac{3x^2 + 2x - 1}{-x^2 + x + 2}$.

1.10. $\lim\limits_{x \to 3} \dfrac{3x^2 - 11x + 6}{2x^2 - 5x - 3}$.

1.11. $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x^2 + x - 6}$.

1.12. $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x^3 + 1}$.

1.13. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x^2 + x - 20}$.

1.14. $\lim\limits_{x \to -3} \dfrac{4x^2 + 11x - 3}{x^2 + 2x - 3}$.

1.15. $\lim\limits_{x \to 3} \dfrac{3x^2 - 7x - 6}{2x^2 - 7x + 3}$.

1.16. $\lim\limits_{x \to -2} \dfrac{4x^2 + 7x - 2}{3x^2 + 8x + 4}$.

1.17. $\lim\limits_{x \to -1} \dfrac{5x^2 + 4x - 1}{3x^2 + x - 2}$.

1.18. $\lim\limits_{x \to -1} \dfrac{x^2 - 4x - 5}{3x^2 + 2x - 2}$.

1.19. $\lim\limits_{x \to -1} \dfrac{7x^2 + 4x - 3}{2x^2 + 3x + 1}$.

1.20. $\lim\limits_{x \to 4} \dfrac{3x^2 - 3x + 2}{x^2 - x - 12}$.

1.21. $\lim\limits_{x \to 2} \dfrac{2x^2 - 9x + 10}{x^2 + 3x - 10}$.

1.22. $\lim\limits_{x \to 1} \dfrac{4x^2 + x - 5}{x^2 - 2x + 1}$.

1.23. $\lim\limits_{x \to 2} \dfrac{-5x^2 + 11x - 2}{3x^2 - x - 10}$.

1.24. $\lim\limits_{x \to 7} \dfrac{x^2 - 5x - 14}{2x^2 - 9x - 35}$.

1.25. $\lim\limits_{x \to 5} \dfrac{3x^2 - 6x - 45}{2x^2 - 3x - 35}$.

1.26. $\lim\limits_{x \to -3} \dfrac{4x^2 + 3x + 15}{x^2 - 6x - 27}$.

1.27. $\lim\limits_{x \to -5} \dfrac{x^2 - 2x - 35}{2x^2 + 11x + 5}$.

1.28. . $\lim\limits_{x \to -8} \dfrac{2x^2 + 15x - 8}{3x^2 + 25x + 8}$.

1.29. $\lim\limits_{x \to 4} \dfrac{3x^2 - 2x - 40}{x^2 - 3x - 4}$.

1.30. $\lim\limits_{x \to -3} \dfrac{2x^2 + 5x - 3}{3x^2 + 10x + 3}$.

2 -TOPSHIRIQ.

2.1. $\lim_{x\to -3} \frac{2x^2+11x+15}{3x^2+5x-12}.$

2.2. $\lim_{x\to 1} \frac{2x^2+5x-10}{x^3-1}.$

2.3. $\lim_{x\to 1} \frac{x^3-3x+2}{x^2-4x+3}.$

2.4. $\lim_{x\to 2} \frac{3x^2+2x+1}{x^3-8}.$

2.5. $\lim_{x\to -1} \frac{x^4-x^2+x+1}{x^4+1}.$

2.6. $\lim_{x\to 1} \frac{2x^2-3x-1}{x^4-1}.$

2.7. $\lim_{x\to 2} \frac{x^2-x+3}{5x^2+3x-3}.$

2.8. $\lim_{x\to -2} \frac{x^2+2x}{x^2+4x+4}.$

2.9. $\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2}.$

2.10. $\lim_{x\to -4} \frac{2x^2+7x-4}{x^3+64}.$

2.11. $\lim_{x\to -5} \frac{4x^2+19x-5}{2x^2+11x+5}.$

2.12. $\lim_{x\to 1} \frac{x^3-x^2+x-1}{x^3+x-2}.$

2.13. $\lim_{x\to 1} \frac{x^2-2x+1}{2x^2-7x+5}.$

2.14. $\lim_{x\to 2} \frac{x^3-8}{2x^2-9x+10}.$

2.15. $\lim_{x\to -2} \frac{9x^2+17x-2}{x^2+2x}.$

2.16. $\lim_{x\to 1} \frac{x^3+x-2}{x^3-x^2-x+1}.$

2.17. $\lim_{x\to 0} \frac{4x^3-2x^2+5x}{3x^2+7x}.$

2.18. $\lim_{x\to 1} \frac{4x^4-5x^2+1}{x^2-1}.$

2.19. $\lim_{x\to 3} \frac{3x^2+5x-1}{x^2-5x+6}.$

2.20. $\lim_{x\to -5} \frac{x^2-x-30}{x^3+125}.$

2.21. $\lim_{x\to 4} \frac{x^2+3x-28}{x^3-64}.$

2.22. $\lim_{x\to 1/2} \frac{8x^3-1}{x^3-1/4}.$

2.23. $\lim_{x\to 4} \frac{x^2+3x-28}{x^2-4x}.$

2.24. $\lim_{x\to -2} \frac{3x^2+11x+10}{x^2-5x+14}.$

2.25. $\lim_{x\to -2} \frac{x^2-4}{3x^2+x-10}.$

2.26. $\lim_{x\to 0} \frac{3x^2+x}{4x^2-5x+1}.$

2.27. $\lim_{x\to 6} \frac{2x^2-11x-6}{3x^2-20x+12}.$

2.28. $\lim_{x\to -6} \frac{x^2+2x-24}{2x^3+15x+18}.$

2.29. $\lim_{x\to 2} \frac{x^3-2x-4}{x^2-11x+18}.$

2.30. $\lim_{x\to 4} \frac{x^3-64}{7x^2-27x-4}.$

3 -TOPSHIRIQ.

3.1. $\lim\limits_{x\to\infty}\dfrac{3x^3-5x^2+2}{2x^3+5x^2-x}$.

3.2. $\lim\limits_{x\to\infty}\dfrac{4x^3+7x}{2x^3-4x^2+5}$.

3.3. $\lim\limits_{x\to\infty}\dfrac{5x^4-3x^2+7}{x^4+2x^3+1}$.

3.4. $\lim\limits_{x\to\infty}\dfrac{7x^3-2x^2+4x}{2x^3+5}$.

3.5. $\lim\limits_{x\to\infty}\dfrac{x^3-4x^2+28x}{5x^3+3x^2+x-1}$.

3.6. $\lim\limits_{x\to\infty}\dfrac{3x^2+10x+3}{2x^2+5x-3}$.

3.7. $\lim\limits_{x\to\infty}\dfrac{-3x^4+x^2+x}{x^4+3x-2}$.

3.8. $\lim\limits_{x\to\infty}\dfrac{2x^2+7x+3}{5x^2-3x+4}$.

3.9. $\lim\limits_{x\to\infty}\dfrac{-x^2+3x+1}{3x^2+x-5}$.

3.10. $\lim\limits_{x\to\infty}\dfrac{x^3-3x^2+10}{7x^3+2x+1}$.

3.11. $\lim\limits_{x\to\infty}\dfrac{4x^2+5x-7}{2x^2-x+10}$.

3.12. $\lim\limits_{x\to\infty}\dfrac{3x^4+2x+1}{x^4-x^3+2x}$.

3.13. $\lim\limits_{x\to\infty}\dfrac{3x^2+2x+9}{2x^2-x+4}$.

3.14. $\lim\limits_{x\to\infty}\dfrac{3x^2+5x-7}{3x^2+x+1}$.

3.15. $\lim\limits_{x\to\infty}\dfrac{2x^3+7x-2}{3x^3-x-4}$.

3.16. $\lim\limits_{x\to 0}\dfrac{18x^2+5x}{8-3x-9x^2}$.

3.17. $\lim\limits_{x\to\infty}\dfrac{3x^4-6x^2+2}{x^4+4x-3}$.

3.18. $\lim\limits_{x\to\infty}\dfrac{8x^2+4x-5}{4x^2-3x+2}$.

3.19. $\lim\limits_{x\to\infty}\dfrac{8x^4-4x^2+3}{2x^4+1}$.

3.20. $\lim\limits_{x\to\infty}\dfrac{3x^2-4x+2}{6x^2+5x+1}$.

3.21. $\lim\limits_{x\to\infty}\dfrac{7x^3+4x}{x^3-3x+2}$.

3.22. $\lim\limits_{x\to\infty}\dfrac{1+4x-x^4}{x+3x^2+2x^4}$.

3.23. $\lim\limits_{x\to\infty}\dfrac{2x^3+7x^2-2}{6x^3-4x+3}$.

3.24. $\lim\limits_{x\to\infty}\dfrac{3x+14x^2}{1+2x+7x^2}$.

3.25. $\lim\limits_{x\to\infty}\dfrac{x-2x^2+5x^4}{2+3x^2+x^4}$.

3.26. $\lim\limits_{x\to\infty}\dfrac{3x^4-2x^2-7}{3x^4+3x+5}$.

3.27. $\lim\limits_{x\to\infty}\dfrac{4-5x^2-3x^5}{x^5+6x+8}$.

3.28. $\lim\limits_{x\to\infty}\dfrac{5x^3-7x^2+3}{2+2x-x^3}$.

3.29. $\lim\limits_{x\to\infty}\dfrac{4x^3-2x+1}{2x^3+3x^2+2}$.

3.30. $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+1}{3x^2+x-5}$.

4 - TOPSHIRIQ.

4.1. $\lim\limits_{x\to-\infty}\dfrac{x^5-2x+4}{2x^4+3x^2+1}$.

4.2. $\lim\limits_{x\to\infty}\dfrac{3x^4+2x-5}{2x^2+x+7}$.

4.3. $\lim\limits_{x\to-\infty}\dfrac{3x^2+7x-4}{x^5+2x-1}$.

4.4. $\lim\limits_{x\to\infty}\dfrac{3x-x^6}{x^2-2x+5}$.

4.5. $\lim\limits_{x\to\infty}\dfrac{2x^3+7x-1}{3x^4+2x+5}$.

4.6. $\lim\limits_{x\to-\infty}\dfrac{2x^3+7x^2+4}{x^4+5x-1}$.

4.7. $\lim\limits_{x\to-\infty}\dfrac{3x^6-5x^2+2}{2x^3+4x-5}$.

4.8. $\lim\limits_{x\to\infty}\dfrac{x^7+5x^2-4x}{3x^2+11x-7}$.

4.9. $\lim\limits_{x\to-\infty}\dfrac{7x^2 5x+9}{1+4x-x^3}$.

4.10. $\lim\limits_{x\to\infty}\dfrac{3x^4+x^2-6}{2x^2+3x+1}$.

4.11. $\lim\limits_{x\to-\infty}\dfrac{2x^2+5x+7}{3x^4-2x^2+x}$.

4.12. $\lim\limits_{x\to\infty}\dfrac{3x^3+4x^2-7x}{2x^2+7x-3}$.

4.13. $\lim\limits_{x\to-1}\dfrac{5x^3-3x^2+7}{2x^4+3x^2+1}$.

4.14. $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+1}{1+2x-x^4}$.

4.15. $\lim\limits_{x\to-\infty}\dfrac{2x^3+3x^2+5}{3x^2-4x+1}$.

4.16. $\lim\limits_{x\to\infty}\dfrac{6x^2-5x+2}{4x^3+2x-1}$.

4.17. $\lim\limits_{x\to-\infty}\dfrac{11x^3+3x}{2x^2-2x+1}$.

4.18. $\lim\limits_{x\to\infty}\dfrac{8x^2+3x+5}{4x^3-2x^2-1}$.

4.19. $\lim\limits_{x\to-\infty}\dfrac{6x^3+5x^2-3}{2x^2-x+7}$.

4.20. $\lim\limits_{x\to\infty}\dfrac{3x^2+4x-7}{x^4-2x^3+1}$.

4.21. $\lim\limits_{x\to-\infty}\dfrac{8x^5+4x^3+3}{2x^3+x-7}$.

4.22. $\lim\limits_{x\to\infty}\dfrac{2x^2-7x+1}{x^3+4x^2-3}$.

4.23. $\lim\limits_{x\to-\infty}\dfrac{5x^4-2x^2+3}{2x^2+3x-7}$.

4.24. $\lim\limits_{x\to\infty}\dfrac{8x^3+x^2-7}{2x^2-5x+3}$.

4.25. $\lim\limits_{x\to-\infty}\dfrac{3x^4+2x^2-8}{8x^3-4x+5}$.

4.26. $\lim\limits_{x\to\infty}\dfrac{3x^4+2x-4}{3x^2-4x+1}$.

4.27. $\lim\limits_{x\to-\infty}\dfrac{7x^3-2x+4}{2x^2+x-5}$.

4.28. $\lim\limits_{x\to\infty}\dfrac{4x^3+5x^2-3x}{3x^2+x+10}$.

4.29. $\lim\limits_{x\to-\infty}\dfrac{2x^2+10x-11}{3x^4-2x+5}$.

4.30. $\lim\limits_{x\to\infty}\dfrac{7x^3+3x-4}{2x^2-5x+1}$.

5 - TOPSHIRIQ.

5.1. $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{7x^3-2x^2+1}$.

5.2. $\lim\limits_{x\to-\infty}\dfrac{3x^2-7x+2}{x^4+2x-4}$.

5.3. $\lim\limits_{x\to\infty}\dfrac{7x^4-3x+4}{3x^2-2x+1}$.

5.4. $\lim\limits_{x\to-\infty}\dfrac{2x^2-x+7}{3x^4-5x^2+10}$.

5.5. $\lim\limits_{x\to-\infty}\dfrac{4x^3-2x^2+x}{3x^2-x}$.

5.6. $\lim\limits_{x\to\infty}\dfrac{3x^4-2x+1}{3x^2+2x-5}$.

5.7. $\lim\limits_{x\to\infty}\dfrac{2x^2-5x+2}{x^4+3x^2-9}$.

5.8. $\lim\limits_{x\to-\infty}\dfrac{5x^2-4x+2}{4x^3+2x-5}$.

5.9. $\lim\limits_{x\to\infty}\dfrac{2x^3+3x^2+2x}{x^2+7x+1}$.

5.10. $\lim\limits_{x\to-\infty}\dfrac{3x^2-7x+5}{4x^5-3x^3+2}$.

5.11. $\lim\limits_{x\to\infty}\dfrac{7x^5+6x^4-x^3}{2x^2+6x+1}$.

5.12. $\lim\limits_{x\to-\infty}\dfrac{4-3x-2x^2}{3x^4+5x}$.

5.13. $\lim\limits_{x\to-\infty}\dfrac{7-3x^4}{2x^3+3x^2-5}$.

5.14. $\lim\limits_{x\to\infty}\dfrac{8x^4+7x^3-3}{3x^2-5x+1}$.

5.15. $\lim\limits_{x\to-\infty}\dfrac{3x+7}{2-3x+4x^2}$.

5.16. $\lim\limits_{x\to-\infty}\dfrac{2x^3-3x+1}{7x+5}$.

5.17. $\lim\limits_{x\to\infty}\dfrac{10x-7}{3x^4+2x^3+1}$.

5.18. $\lim\limits_{x\to-\infty}\dfrac{5x^4-3x^2}{1+2x+3x^2}$.

5.19. $\lim\limits_{x\to\infty}\dfrac{5x+3}{x^3-4x^2-x}$.

5.20. $\lim\limits_{x\to-\infty}\dfrac{3x^4+5x}{2x^2-3x-7}$.

5.21. $\lim\limits_{x\to\infty}\dfrac{2x^2-5x+3}{3x^4-2x^2+x}$.

5.22. $\lim\limits_{x\to-\infty}\dfrac{2x^5-x^3}{4x^2+3x-6}$.

5.23. $\lim\limits_{x\to\infty}\dfrac{3x+1}{x^3-2x^2+x}$.

5.24. $\lim\limits_{x\to-\infty}\dfrac{2-x-3x^2}{x^3-16}$.

5.25. $\lim\limits_{x\to\infty}\dfrac{4x^2-10x+7}{2x^3-3x}$.

5.26. $\lim\limits_{x\to-\infty}\dfrac{2x^3-3x+1}{x^5+4x^3}$.

5.27. $\lim\limits_{x\to\infty}\dfrac{2x-13}{x^7-3x^5-4x}$.

5.28. $\lim\limits_{x\to-\infty}\dfrac{2x^2-3x+1}{x^2+2x^2+5}$.

5.29. $\lim\limits_{x\to\infty}\dfrac{x^3-81}{3x^2+4x+2}$.

5.30. $\lim\limits_{x\to-\infty}\dfrac{7x+4}{3x^3-5x+1}$.

6 -TOPSHIRIQ.

6.1. $\lim\limits_{x\to 3}\dfrac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}}.$

6.2. $\lim\limits_{x\to -4}\dfrac{\sqrt{x+12}-\sqrt{4-x}}{x^2+2x-8}.$

6.3. $\lim\limits_{x\to -3}\dfrac{\sqrt{x+10}-\sqrt{4-x}}{2x^2-x-21}.$

6.4. $\lim\limits_{x\to -2}\dfrac{\sqrt{2-x}-\sqrt{x+6}}{x^2-x-6}.$

6.5. $\lim\limits_{x\to 1}\dfrac{\sqrt{3+2x}-\sqrt{x+4}}{3x^2-4x+1}.$

6.6. $\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{\sqrt{5-x}-\sqrt{x+1}}.$

6.7. $\lim\limits_{x\to -1}\dfrac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}.$

6.8. $\lim\limits_{x\to 4}\dfrac{2x^2-9x+4}{\sqrt{5-x}-\sqrt{x-3}}.$

6.9. $\lim\limits_{x\to 5}\dfrac{\sqrt{2x+1}-\sqrt{x+6}}{2x^2-7x-15}.$

6.10. $\lim\limits_{x\to -5}\dfrac{\sqrt{3x+17}-\sqrt{2x+12}}{x^2+8x+15}.$

6.11. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+2}-\sqrt{2}}{\sqrt{x^2+1}-1}.$

6.12. $\lim\limits_{x\to 0}\dfrac{\sqrt{7-x}-\sqrt{7+x}}{\sqrt{7}x}.$

6.13. $\lim\limits_{x\to 0}\dfrac{3x}{\sqrt{1+x}-\sqrt{1-x}}.$

6.14. $\lim\limits_{x\to 4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}.$

6.15. $\lim\limits_{x\to -1}\dfrac{\sqrt{5+x}-2}{\sqrt{8-x}-3}.$

6.16. $\lim\limits_{x\to 5}\dfrac{\sqrt{x+4}-3}{\sqrt{x-1}-2}.$

6.17. $\lim\limits_{x\to 7}\dfrac{\sqrt{x-3}-2}{\sqrt{x+2}-3}.$

6.18. $\lim\limits_{x\to 3}\dfrac{\sqrt{4x-3}-3}{x^2-9}.$

6.19. $\lim\limits_{x\to 3}\dfrac{\sqrt{5x+1}-4}{x^2+2x-15}.$

6.20. $\lim\limits_{x\to 0}\dfrac{2-\sqrt{x^2+4}}{3x^2}.$

6.21. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{x^2+16}-4}.$

6.22. $\lim\limits_{x\to 0}\dfrac{3x}{\sqrt{5-x}-\sqrt{5+x}}.$

6.23. $\lim\limits_{x\to 9}\dfrac{\sqrt{2x+7}-5}{3-\sqrt{x}}.$

6.24. $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{\sqrt{6x+1}-5}.$

6.25. $\lim\limits_{x\to 3}\dfrac{x^3-27}{\sqrt{3x}-x}.$

6.26. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-1}{x^3+x^2}.$

6.27. $\lim\limits_{x\to -4}\dfrac{\sqrt{x+20}-4}{x^3+64}.$

6.28. $\lim\limits_{x\to 1}\dfrac{3x^2-2}{\sqrt{8+x}-3}.$

6.29. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+x}-3}{x^2+x}.$

6.30. $\lim\limits_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x^3-8}.$

7 - TOPSHIRIQ.

7.1. $\lim_{x\to\infty}\left(\frac{x+4}{x+8}\right)^{-3x}$

7.2. $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{2x-3}$.

7.3. $\lim_{x\to\infty}\left(\frac{2x}{1+2x}\right)^{-4x}$

7.4. $\lim_{x\to\infty}\left(\frac{x-1}{x}\right)^{2-3x}$.

7.5. $\lim_{x\to\infty}\left(\frac{2x+5}{2x+1}\right)^{5x}$

7.6. $\lim_{x\to\infty}\left(\frac{x+3}{x}\right)^{-5x}$.

7.7. $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{1+2x}$

7.8. $.\lim_{x\to\infty}\left(\frac{x+3}{x-1}\right)^{x-4}$.

7.9. $\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{3x}$

7.10. $\lim_{x\to\infty}\left(\frac{x-7}{x}\right)^{2x+1}$.

7.11. $\lim_{x\to\infty}\left(\frac{x-1}{x+4}\right)^{3x+2}$

7.12. $\lim_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{x+2}$.

7.13. $\lim_{x\to\infty}\left(\frac{x-2}{x+1}\right)^{2x-3}$

7.14. $\lim_{x\to\infty}\left(\frac{x}{x-3}\right)^{x-5}$.

7.15. $\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{2x}$

7.16. $\lim_{x\to\infty}\left(\frac{2x-1}{2x+4}\right)^{3x-1}$.

7.17. $\lim_{x\to\infty}\left(\frac{2x-4}{2x}\right)^{-3x}$

7.18. $\lim_{x\to\infty}\left(\frac{x+5}{x}\right)^{3x+4}$.

7.19. $\lim_{x\to\infty}\left(\frac{x-7}{x+1}\right)^{4x-2}$

7.20. $\lim_{x\to\infty}\left(\frac{x+2}{x}\right)^{3-2x}$.

7.21. $\lim_{x\to\infty}\left(\frac{2-3x}{5-3x}\right)^{x}$

7.22. $\lim_{x\to\infty}\left(\frac{1-x}{2-x}\right)^{3x}$.

7.23. $\lim_{x\to\infty}\left(\frac{4x-1}{4x+1}\right)^{2x}$

7.24. $\lim_{x\to\infty}\left(\frac{3x+4}{3x}\right)^{-2x}$.

7.25. $\lim_{x\to\infty}\left(\frac{2x-1}{2x+4}\right)^{-x}$

7.26. $\lim_{x\to\infty}\left(\frac{3x+4}{3x+5}\right)^{x+1}$.

7.27. $\lim_{x\to\infty}\left(\frac{1+2x}{3+2x}\right)^{-x}$

7.28. $\lim_{x\to\infty}\left(\frac{3x}{3x+2}\right)^{x-2}$.

7.29. $\lim_{x\to\infty}\left(\frac{x}{x-1}\right)^{3-2x}$

7.30. $\lim_{x\to\infty}\left(\frac{4-2x}{1-2x}\right)^{x+1}$.

8 -TOPSHIRIQ

8.1. $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{3x^2}.$

8.2. $\lim\limits_{x\to 0}\dfrac{\sin 3x-\sin x}{5x}.$

8.3. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 5x}{2x^2}.$

8.4. $\lim\limits_{x\to 0}\dfrac{tg 3x}{2\sin x}.$

8.5. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{3x^2}.$

8.6. $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{\sin 3x}.$

8.7. $\lim\limits_{x\to 1}(1-x)tg\dfrac{\pi x}{2}.$

8.8. $\lim\limits_{x\to \pi/2}\dfrac{1-\sin x}{\pi-2x}.$

8.9. $\lim\limits_{x\to 0}\dfrac{tg 2x-\sin 2x}{x^2}.$

8.10. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{xtgx}.$

8.11. $\lim\limits_{x\to 0}\left(\dfrac{1}{tgx}-\dfrac{1}{\sin x}\right).$

8.12. $\lim\limits_{x\to 0}\dfrac{\sin^2 3x-\sin^2 x}{x^2}.$

8.13. $\lim\limits_{x\to 0}\dfrac{\sin 7x+\sin 3x}{x\sin x}.$

8.14. $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{2x^2}.$

8.15. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 4x}{3x^2}.$

8.16. $\lim\limits_{x\to 0}\dfrac{arctg 2x}{tg 3x}.$

8.17. $\lim\limits_{x\to 0}\dfrac{tg 3x-\sin 3x}{2x^2}.$

8.18. $\lim\limits_{x\to \pi/4}\dfrac{1-\sin 2x}{\pi-4x}.$

8.19. $\lim\limits_{x\to 0}\dfrac{\cos 4x-\cos^3 4x}{3x^2}.$

8.20. $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin 2x}-\dfrac{1}{tg 2x}\right).$

8.21. $\lim\limits_{x\to 0}\dfrac{\cos^2 x-\cos^2 2x}{x^2}.$

8.22. $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{x^2-x}.$

8.23. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 2x}{x\arcsin x}.$

8.24. $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{x\sin x}.$

8.25. $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos x}{4x^2}.$

8.26. $\lim\limits_{x\to 0}\dfrac{\sin 5x+\sin x}{\arcsin x}.$

8.27. $\lim\limits_{x\to \pi/2}\dfrac{1-\sin x}{(\pi/2-x)^2}.$

8.28. $\lim\limits_{x\to \pi/2}(\pi/2-x)tgx.$

8.29. $\lim\limits_{x\to 0}\dfrac{7x}{\sin x+\sin 7x}.$

8.30. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^3 x}{5x^2}.$

NAMUNAVIY VARIANTNI YECHISH

Ko‘rsatilgan limitlarni toping

1. $\lim\limits_{x\to -2}\frac{5x^2+13x+6}{3x^2+2x-8}$.

Yechish:

$$\lim\limits_{x\to -2}\frac{5x^2+13x+6}{3x^2+2x-8}=\lim\limits_{x\to -2}\frac{(x+2)(5x+3)}{(x+2)(3x-4)}=\lim\limits_{x\to -2}\frac{5x+3}{3x-4}=\frac{7}{10}=0{,}7$$

2. $\lim\limits_{x\to 4}\frac{3x^2-10x-8}{4x^2+6x-64}$.

Yechish:

$$\lim\limits_{x\to 4}\frac{3x^2-10x-8}{4x^2+6x-64}=\frac{0}{24}=0$$

3. $\lim\limits_{x\to \infty}\frac{7x^4+2x^3+5}{6x^4+3x^2-7x}$.

Yechish:

$$\lim\limits_{x\to \infty}\frac{7x^4+2x^3+5}{6x^4+3x^2-7x}=\lim\limits_{x\to \infty}\frac{x^4(7+2/x+5/x^4)}{x^4(6+3/x^2-7/x^3)}=\frac{7}{6}$$

4. $\lim\limits_{x\to -\infty}\frac{10x-3}{2x^3+4x+3}$.

Yechish:

$$\lim\limits_{x\to -\infty}\frac{10x-3}{2x^3+4x+3}=\lim\limits_{x\to -\infty}\frac{x(10-3/x)}{x^3(2+4/x^2+3/x^3)}=\lim\limits_{x\to -\infty}\frac{10-3/x}{x^2(2+4/x^2+3/x^3)}=\frac{10}{\infty}=0.$$

5. $\lim\limits_{x\to -\infty}\frac{2x^5+3x^3-4x}{3x^2-4x+2}$.

Yechish:

$$\lim\limits_{x\to -\infty}\frac{2x^5+3x^3-4x}{3x^2-4x+2}=\lim\limits_{x\to -\infty}\frac{x^5(2+3/x^2-4/x^4)}{x^2(3-4/x+2/x^2)}= \quad =\lim\limits_{x\to -\infty}\frac{x^3(2+3/x^2-4/x^4)}{3-4/x+2/x^2}=\frac{-\infty}{3}=-\infty.$$

6. $\lim\limits_{x\to 4}\frac{\sqrt{21+x}-5}{x^3-64}$.

Yechish:

$$\lim\limits_{x\to 4}\frac{\sqrt{21+x}-5}{x^3-64}=\lim\limits_{x\to 4}\frac{(\sqrt{21+x}-5)(\sqrt{21+x}+5)}{(x^3-64)(\sqrt{21+x}+5)}=\lim\limits_{x\to 4}\frac{21+x-25}{(x-4)(x^2+4x+16)(\sqrt{21+x}+5)}=$$

$$=\lim_{x\to4}\frac{x-4}{(x^3-64)(\sqrt{21+x}+5)}=\lim_{x\to4}\frac{1}{(x^2-4x+16)(\sqrt{21+x+5})}=\frac{1}{480}.$$

7. $\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{2-5x}$.

Yechish:

$$\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{2-5x}=\lim_{x\to\infty}\left(1+\frac{2x}{2x-3}-1\right)^{2-5x}=\lim_{x\to\infty}\left(1+\frac{2x-2x+3}{2x-3}\right)^{2-5x}=\lim_{x\to\infty}\left(1+\frac{3}{2x-3}\right)^{2-5x}=$$

$$=\lim_{x\to\infty}\left(\left(1+\frac{3}{2x-3}\right)^{(2x-3)/3}\right)^{3(2-5x)/(2x-3)}=\lim_{x\to\infty}e^{3(2-5x)/(2x-3)}=e^{-15/2}.$$

8. $\lim_{x\to\pi}\frac{1-\sin(x/2)}{\pi^2-x^2}$.

Yechish:

$$\lim_{x\to\pi}\frac{1-\sin(x/2)}{\pi^2-x^2}=\lim_{x\to\pi}\frac{1-\cos(\pi/2-x/2)}{\pi^2-x^2}=\lim_{x\to\pi}\frac{2\sin^2\big((\pi-x)/4\big)}{(\pi-x)(\pi+x)}=$$

$$=\lim_{x\to\pi}\frac{2\sin\big((\pi-x)/4\big)\sin\big((\pi-x)/4\big)}{4\cdot\frac{\pi-x}{4}(\pi+x)}==\frac{1}{2}\lim_{x\to\pi}\frac{\sin\big((\pi-x)/4\big)}{\pi+x}=\frac{1}{2}\frac{0}{2\pi}=0.$$

V. BIR O‘ZGARUVCHILI FUNKSIYANING DIFFERENSIAL HISOBI VA UNING TADBIQLARI

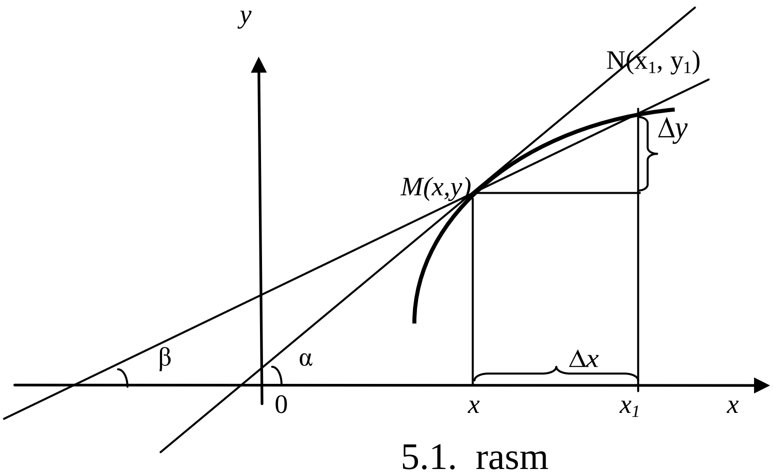
5.1. HOSILA, UNING GEOMETRIK VA FIZIK MA’NOSI. DIFFERENSIALLASH QOIDALARI VA FORMULALARI

Eslatib o‘tamizki, $y = f(x)$ funksiyaning *orttirmasi* deb

$$\Delta y = f(x+\Delta x) - f(x)$$

ayirmaga aytiladi, bu yerda Δx argument x ning orttirmasi. 5.1 rasmdan ko‘rinib turibdiki

$$\frac{\Delta y}{\Delta x} = tg\beta \quad . \tag{5.1}$$



5.1. rasm

Funksiya orttirmasi Δy ning, argument orttirmasi Δx ga nisbatining Δx ixtiyoriy tarzda nolga intilgandagi limitiga, $y = f(x)$ *funksiyaning x nuqtadagi hosilasi* deyiladi va quyidagilarning biri orqali belgilanadi: $y', f'(x), \frac{dy}{dx}$. Shunday qilib, ta’rifga ko‘ra

$$y' = f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad . \tag{5.2}$$

Agar yuqoridagi (5.2) formulada limit mavjud bo‘lsa, $f(x)$ funksiya *x nuqtada differensiallanuvchi,* hosila olish amalini esa d*ifferensiallash* deyiladi.

(5.1) tenglik va hosila ta'rifidan, hosilaning x nuqtadagi qiymati $f(x)$ funksiya grafigiga *M*(*x*,*y*) nuqtada o'tkazilgan urinmaning *Ox* o'qi musbat yo'nalishi bilan hosil qilgan burchagi tangensiga tengligi kelib chiqadi.

Osongina ko'rsatish mumkinki, fizik jihatdan qaraganda, $y' = f'(x)$ hosilaning qiymati x argumentga nisbatan funksiya o'zgarishining tezligiga teng.

Agar C- o'zgarmas son bo'lsa va $u = u(x), v = v(x)$ differensiallanuvchi funksiyalar bo'lsa, u holda quyidagi differensiallash formulalari o'rinli:

1) $C' = 0$;

2) $x' = 1$;

3) $(u \pm v)' = u' \pm v'$;

4) $(Cu)' = Cu'$;

5) $(uv)' = u'v + uv'$;

6) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, \quad v \neq 0$;

7) $\left(\frac{C}{v}\right)' = -\frac{Cv'}{v^2}, \quad v \neq 0$;

8) agar $y = f(u)$, $u = \varphi(x)$, ya'ni $y = f(\varphi(x))$ murakkab funksiya differensiallanuvchi funksiyalardan tuzilgan bo'lsa, u holda

$$y_x' = y_u' u_x' \text{ yoki } \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

9) agar $y = f(x)$ funksiyaning, $x = g(y)$ va $\frac{dg}{dy} = g'(y) \neq 0$ Differensiallanuvchi teskari funksiyasi mavjud bo'lsa, u holda

$$f'(x) = \frac{1}{g'(y)}.$$

Hosilaning ta'rifi va differensiallash qoidalariga asosan *asosiy elementar funksiyalarning hosilalari* jadvalini tuzish mumkin:

1) $\left(u^{\alpha}\right)'=\alpha u^{\alpha-1}u'\ (\alpha\in\mathbf{R})$;

2) $\left(a^{u}\right)'=a^{u}\cdot\ln a\cdot u'$;

3) $\left(e^{u}\right)'=e^{u}\cdot u'$;

4) $(\log_a u)'=\dfrac{u'}{u\cdot\ln a}$;

5) $(\ln u)'=\dfrac{u'}{u}$;

6) $(\sin u)'=\cos u\cdot u'$;

7) $(\cos u)'=-\sin u\cdot u'$;

8) $(tgu)'=\dfrac{u'}{\cos^2 u}$;

9) $(ctgu)'=-\dfrac{u'}{\sin^2 u}$;

10) $(\arcsin u)'=\dfrac{u'}{\sqrt{1-u^2}}$;

11) $(\arccos u)'=-\dfrac{u'}{\sqrt{1-u^2}}$;

12) $(arctgu)'=\dfrac{u'}{1+u^2}$;

13) $(arcctgu)'=-\dfrac{u'}{1+u^2}$;

14) $(shu)'=chu\cdot u'$;

15) $(chu)'=shu\cdot u'$;

16) $(thu)'=\dfrac{u'}{ch^2u}$;

17) $(cthu)'=-\dfrac{u'}{sh^2u}$.

$y=f(x)$ egri chiziqqa $M_0(x_0, f(x_0))$ nuqtada o‘tkazilgan *urinma tenglamasi*

$$y-f(x_0)=f'(x_0)(x-x_0)\qquad \left(f'(x_0)\neq 0\right).$$

$y=f(x)$ egri chiziqqa $M_0(x_0, f(x_0))$ nuqtada o‘tkazilgan *normal tenglamasi*

$$y-f(x_0)=-\frac{1}{f'(x_0)}(x-x_0)\qquad \left(f'(x_0)\neq 0\right).$$

$f'(x_0)=0$ da normal tenglamasi $x=x_0$ ko‘rinishda bo‘ladi.

Ikki egri chiziqning, ular kesishish nuqtasida hosil qilgan burchagi deb, shu nuqtada ularga o‘tkazilgan urinmalar orasidagi burchakka aytiladi.

1-misol. Hosila ta’rifidan foydalanib $y=\dfrac{2x}{3x+1}$ funksiyaning hosilasini toping.

Yechish: Ta’rifga asosan ixtiyoriy Δx da:

$$\Delta y=\frac{2(x+\Delta x)}{3(x+\Delta x)+1}-\frac{2x}{3x+1}=\frac{6x^2+6x\Delta x+2x+2\Delta x-6x^2-6x\Delta x-2x}{(3(x+\Delta x)+1)(3x+1)}=\frac{2\Delta x}{(3(x+\Delta x)+1)(3x+1)}$$

$\frac{\Delta y}{\Delta x}=\frac{2}{(3(x+\Delta x)+1)(3x+1)}$ ekanligidan $y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{2}{(3(x+\Delta x)+1)(3x+1)}=\frac{2}{(3x+1)^2}$

kelib chiqadi.

2-misol. $y=|x|$ funksiyaning $x=0$ nuqtadagi hosilasi qiymatini toping.

Yechish: Argumentning $x=0$ nuqtadagi ixtiyoriy Δx orttirmasi uchun funksiya orttirmasi

$$\Delta y=|\Delta x|=\begin{cases}-\Delta x, агар.\Delta x<0\\ \Delta x, агар.\Delta x>0\end{cases}.$$

Hosilaning ta'rifiga ko'ra esa

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\begin{cases}-1, агар.\Delta x<0\\ 1, агар.\Delta x>0\end{cases}.$$

Bu esa $y=|x|$ funksiya $x=0$ nuqtada hosilaga ega emasligini ko'rsatadi. Shuni ta'kidlash lozimki, funksiya $x=0$ nuqtada uzluksiz, chunki

$$\lim_{\Delta x\to 0}|\Delta y|=\lim_{\Delta x\to 0}|\Delta x|=0$$

Shunday qilib, funksiya biror nuqtada uzluksiz ekanligidan, uning Differensiallanuvchi ekanligi kelib chiqmaydi. Osongina ko'rsatish mumkinki, funksiyaning Differensiallanuvchi bo'lgan barcha nuqtalarida uzluksiz bo'ladi.

5.2 LOGARIFMIK DIFFERENSIALLASH

$y=f(x)$ funksiyaning *logarifmik hosilasi* deb, bu funksiyaning logarifmidan olingan hosilaga aytiladi, ya'ni

$$(\ln f(x))'=\frac{f'(x)}{f(x)}$$

Funksiyani logarifmlash va differensiallashning ketma- ket qo'llanilishi *logarifmik differensiallash* deb ataladi. Ba'zi hollarda avval funksiyani logarifmlash, uning hosilasini topishni osonlashtiradi. Masalan, $y = u^v$, (bu yerda $u = u(x), v = v(x)$) funksiyaning hosilasini topishda, avval logarifmlash quyidagi formulaga olib keladi:

$y' = u^v \ln u \cdot v' + v u^{v-1} \cdot u'$.

1-misol. $y = (\sin 2x)^{x^3}$ funksiyaning hosilasini toping.

Yechish: Berilgan funksiyani logarifmlasak,

$\ln y = x^3 \ln \sin 2x$.

Tenglikning ikkala tomonini *x* bo‘yicha differensiallab

$(\ln y)' = (x^3)' \cdot \ln \sin 2x + x^3 (\ln \sin 2x)'$

ni hosil qilamiz. Bu yerdan

$\frac{y'}{y} = 3x^2 \cdot \ln \sin 2x + x^3 \frac{1}{\sin 2x} 2\cos 2x$ yoki $y' = y(3x^2 \cdot \ln \sin 2x + 2x^3 ctg\, 2x)$,

$y' = (\sin 2x)^{x^3} (3x^2 \cdot \ln \sin 2x + 2x^3 ctg\, 2x)$.

Agar *y* va *x* o‘zgaruvchilar orasidagi bog‘lanish oshkormas ko‘rinishda bo‘lib, $F(x, y) = 0$ tenglama bilan berilgan bo‘lsa, u holda $y' = y'_x$ hosilani topish uchun eng sodda hollarda *y* ni *x* ning funksiyasi deb qarab $F(x, y) = 0$ tenglamaning ikkala tomonini differensiallab, hosil bo‘lgan chiziqli tenglamadan y' ni topish mumkin.

2-misol. Agar $x^3 + y^3 - 3xy = 0$ bo‘lsa, y' toping.

Yechish: *y* ni *x* ning funksiyasi deb hisoblab, berilgan tenglamaning ikkala tomonini differensiallaymiz:

$3x^2 + 3y^2 y' - 3y - 3xy' = 0$.

Bu yerdan

$y' = \frac{3x^2 - 3y}{3x - 3y^2}$.

5.3. YUQORI TARTIBLI HOSILALAR

Biror u=f(x) funksiyaning ikkinchi tartibli hosilasi deb, uning birinchi tartibli hosilasidan olingan hosilaga aytiladi, ya'ni $(y')'$ va uni y'' va $f''(x)$ yoki $\frac{d^2y}{dx^2}$ kabi belgilarning biri orqali yoziladi. Agar moddiy nuqtaning to'g'ri chiziqli harakati $S=S(t)$ funksiya bilan berilgan bo'lsa, $S'=S'(t)$ va $S''=S''(t)$ lar mos ravishda uning tezligi va tezlanishini ifodalaydi.

Agar u funksiyaning x argumentga bo'lgan bog'lanishi $x=x(t)$, $u=u(t)$ parametrik tenglamalar bilan berilgan bo'lsa, u holda:

$$\frac{dy}{dx}=\frac{y'(t)}{x'(t)}, \qquad \frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{y'(t)}{x'(t)}\right)\cdot\frac{1}{x'(t)}, \qquad (5.3)$$

bu yerda shtrix t bo'yicha hosilani bildiradi.

$u=f(x)$ *funksiyaning n-tartibli hosilasi* deb uning (n-1)- tartibli hosilasidan olingan hosilaga aytiladi. Uni belgilash uchun quyidagi belgilarning biri ishlatiladi: $y^{(n)}$, $f^{(n)}(x)$, $\frac{d^n y}{dx^n}$. Demak, $y^{(n)}=(y^{(n-1)})'$ ekan.

1-misol. $y=\ln\left(x+\sqrt{x^2+a^2}\right)$ funksiyaning 2- tartibli hosilasi hisoblansin.

Yechish:

$$y'=\frac{\left(x+\sqrt{x^2+a^2}\right)'}{x+\sqrt{x^2+a^2}}=\frac{1}{x+\sqrt{x^2+a^2}}\cdot\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)=\frac{\sqrt{x^2+a^2}+x}{\left(x+\sqrt{x^2+a^2}\right)\cdot\sqrt{x^2+a^2}}=\frac{1}{\sqrt{x^2+a^2}},$$

$$y''=-\frac{1}{2}\left(x^2+a^2\right)^{\frac{-3}{2}}\cdot 2x=-\frac{x}{\sqrt{\left(x^2+a^2\right)^3}}.$$

2-misol. $y=(2x-1)^4$ funksiya uchun $y'(1)$ va $y'(-1)$ hamda $y''(1)$, $y''(-1)$ hisoblansin.

Yechish: $y'=8(2x-1)^3$ bo'lganligidan $y'(1)=8$, $y'(-1)=-216$, $y''=48(2x-1)^2$ dan esa, $y''=(+1)=48$ va $y''=(-1)=432$.

3-misol. $y=\sin x$ funksiyaning n- tartibli hosilasi hisoblansin.

Yechish: $y' = \cos x = \sin\left(x + \frac{\pi}{2}\right)$,

$y'' = \cos\left(x + \frac{\pi}{2}\right) = \sin\left(x + 2 \cdot \frac{\pi}{2}\right)$,

$y''' = \cos\left(x + 2 \cdot \frac{\pi}{2}\right) = \sin\left(x + 3 \cdot \frac{\pi}{2}\right)$,

....

$y^{(n)} = \cos\left(x + (n-1)\frac{\pi}{2}\right) = \sin\left(x + n\frac{\pi}{2}\right)$.

4- misol. $x=\ln t$ va $y=t^3+2t+1$ parametrik tenglamalar bilan berilgan funksiyaning 2- tartibli hosilasi hisoblansin.

Yechish: (5.3) formulaga binoan, $\frac{dy}{dx} = \frac{3t^2+2}{1/t} = 3t^3 + 2t,$

$\frac{d^2y}{dx^2} = \frac{9t^2+2}{1/t} = 9t^3 + 2t$.

5.4. BIRINCHI VA YUQORI TARTIBLI DIFFERENSIALLAR HAMDA ULARNING TADBIQLARI

Biror $y=f(x)$ *funksiyaning ixtiyoriy x nuqtadagi 1-tartibli differensiali* deb, uning shu nuqtadagi orttirmasining, $\Delta x=dx$ orttirmaga nisbatan chiziqli bo‘lgan qismiga aytiladi. Umuman, funksiyaning differensiali *du* ni hisoblash uchun uning *x* nuqtadagi hosilasini *dx* ga ko‘paytiriladi. $dy=y'dx=f''(x)dx$. Shu boisdan, $y' = \frac{dy}{dx}$ tenglik har doim o‘rinlidir. 5.2-rasmdan ko‘rinib turibdiki, agar *MN*, $y=f(x)$ funksiya grafiginining yoyi bo‘lib, *MT* esa, uning $M(x,y)$ nuqtasiga o‘tkazilgan urinmasi hamda $AB=\Delta x=dx$ kabi bo‘ladigan bo‘lsa, u holda $CT=dy$ va $CN=\Delta y$ kabi bo‘ladi. Funksiyaning *dy* differensiali o‘zining Δy orttirmasidan, Δx ga nisbatan yuqori tartibdagi cheksiz kichik miqdorgagina farq qiladi.

Differensialning ta’rifidan hamda hosilani hisoblash qoidalariga ko‘ra, $u=u(x)$ va $v=v(x)$ funksiyalar uchun quyidagilarni yozish mumkin:

1) $dC=0$ (C=const.);

2) $dx=\Delta x$ (bu yerda x erkli o‘zgaruvchi);

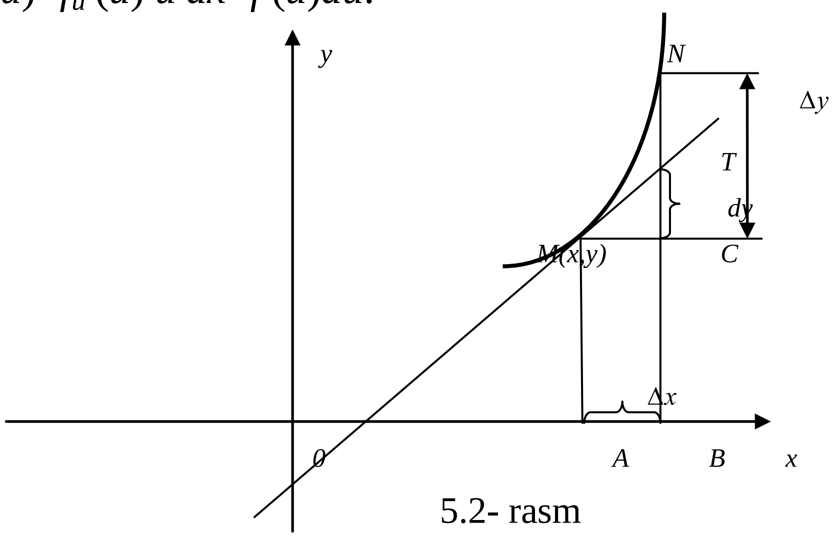
3) $d(u\pm v)=du\pm dv$;

4) $d(uv)=udv+vdu$;

5) $d(Cu)=Cdu$;

6) $d(u/v)=(vdu-udv)/v^2$ $(v\neq 0)$;

7) $df(u)=f_u'(u)\,u'dx=f'(u)du$.



5.2- rasm

1-misol. $y=sin^5 3x$ ning differensiali hisoblansin.

Yechish: Funksiyaning hosilasi $y'=5sin^4 3x\cdot sos3x\cdot 3$ bo‘lganligidan, quyidagini hosil qilamiz $dy=15sin^4 3x\cdot sos3xdx$.

$y=f(x)$ funksiyaning n-tartibli differensiali deb, uning (n-1)- tartibli differensialidan hisoblangan differensialiga aytiladi, ya’ni, $d^n y=d(d^{n-1}y)$. Ta’rifga binoan $d^2y=y''dx^2$, $d^3y=y'''dx^3$,….,$d^ny=y^{(n)}dx^n$ bo‘ladi. ($dx^n=(dx)^n$).

Agar $y=f(u)$ va $u=\varphi(x)$ bo‘lsa, u holda: $d^2y=y''(du)^2+y'\,d^2u$, bu yerda, differensiallash u o‘zgaruvchiga nisbatan bajariladi.

2-misol. $y=ln(1+x^2)$ funksiya uchun d^2y hisoblansin.

$y' = \frac{2x}{1+x^2}$ va $y'' = \frac{2(1-x^2)}{(1+x^2)^2}$ bo‘lganligidan, $d^2y = \frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}dx^2$.

Yuqorida ta’kidlaganidek, $\Delta y \approx dy$ yoki $f(x+\Delta x)-f(x) \approx f'(x)dx$ deb yozish mumkin, bundan esa, $f(x+\Delta x) \approx f(x)+f'(x)dx$ ni hosil qilamiz.

Ushbu formula, ko‘pincha, $f(x)$ funksiyaning qiymatini taqribiy hisoblash uchun qo‘llaniladi (bu yerdagi Δx argument x ning kichik orttirmasidir).

3-misol. Agar kubning hajmi 27 birlikdan 27,1m^3 gacha ortgan bo‘lsa, uning tomonlarining orttirmasi hisoblansin.

Yechish: Agar kubning hajmini x deb belgilasak, u holda uning tomonlari $y = \sqrt[3]{x}$ ga teng bo‘ladi. Masalaning shartiga ko‘ra, x=27, Δx =0,1 bo‘lganligi uchun kub tomonlarining orttirmasi $\Delta y \approx dy = y'(x)\cdot \Delta x = \frac{1}{3\sqrt[3]{27^2}}\cdot 0{,}1 = \frac{0{,}1}{27} \approx 0{,}0037$ ga teng bo‘ladi.

5.5. Lopital qoidalari.

1-teorema (Roll). *Agar u=f(x) funksiya,* [*a;b*] *kesmada uzluksiz bo‘lib, kesmaning ichidagi barcha nuqtalarda differensiallanuvchi hamda f(a)=f(b) bo‘lsa, u holda hech bo‘lmaganda Shunday bir x=s nuqta (a<s<b) mavjudki, har doim uning uchun f '(c)=0 shart o‘rinli bo‘ladi*.

2- teorema (Lagranj). *Agar u=f(x) funksiya,* [*a;b*] *kesmada uzluksiz bo‘lib, kesmaning ichidagi barcha nuqtalarda differensiallanuvchi bo‘lsa, u holda hech bo‘lmaganda Shunday bir x=s nuqta (a<s<b) mavjud bo‘ladiki, har doim uning uchun f(b)-f(a)=f'(c)(b-a) formula o‘rinli bo‘ladi*.

Ushbu formulani *chekli orttirmalar haqidagi Lagranj formulasi* deb yuritiladi.

3- teorema (Koshi). *Agar u=f(x) bilan u=φ(x) funksiyalar,* [*a*; *b*] *kesmada uzluksiz hamda shu kesma ichida differensiallanuvchi (φ'(x)≠0) bo'lsalar, u holda hech bo'lmaganda Shunday bir x=s nuqta mavjud bo'ladiki (a<s<v), har doim quyidagi tenglik o'rinli bo'ladi:*

$$\frac{f(b)-f(a)}{\phi(b)-\phi(a)}=\frac{f'(c)}{\phi'(c)}, \varphi'(c)\neq 0$$

Lopital qoidasi ($\frac{0}{0}$ ***va*** $\frac{\infty}{\infty}$ ***kabi aniqmasliklarni ochish).*** *Agar u=f(x) bilan u=φ(x) funksiyalar, biror* $x=x_0$ *nuqta atrofida Koshi teoremasining shartlarini qanoatlantirib,* $x\to x_0$ *da 0 ga yoki ±∞ ga intiladigan bo'lsalar hamda* $\lim_{x\to x_0}\frac{f'(x)}{\varphi'(x)}$ *chekli limit mavjud bo'lsa, u holda* $\lim_{x\to x_0}\frac{f(x)}{\varphi(x)}$ *chekli limit mavjud bo'ladi va bu limitlar o'zaro teng bo'ladilar, ya'ni:* $\lim_{x\to x_0}\frac{f(x)}{\varphi(x)}=\lim_{x\to x_0}\frac{f'(x)}{\varphi'(x)}$.

Ushbu qoida $x_0=\pm\infty$ bo'lganda ham o'rinli bo'lib qolaveradi.

Agar $\frac{f'(x)}{\varphi'(x)}$ nisbat uchun qaralayotgan nuqtada yana yuqoridagi aniqmasliklar yuz beradigan bo'lsa, u holda, agar *f'(x)* va *φ'(x)* lar uchun *f(x)* bilan *φ(x)* larga nisbatan yuqorida keltirilgan shartlar o'rinli bo'ladigan bo'lsa, ikkinchi tartibli hosilalarning nisbati uchun ham limitga o'tish mumkin va hokazo. Ta'kidlash lozimki, garchi funksiyalar nisbatining limiti mavjud bo'lsada, ammo hosilalar nisbatining hech qanday limiti mavjud bo'lmasligi ham mumkin.

1-misol. $\lim_{x\to\infty}\frac{x+\sin x}{x+\cos x}$ ni hisoblansin.

Yechish: $\lim_{x\to\infty}\frac{x+\sin x}{x+\cos x}=\lim_{x\to\infty}\frac{1+\frac{\sin x}{x}}{1+\frac{\cos x}{x}}=1.$

Ammo, $\lim\limits_{x\to\infty}\frac{(x+\sin x)'}{(x+\cos x)'}=\lim\limits_{x\to\infty}\frac{1+\cos x}{1-\sin x}$ ning hech qanaqa limiti mavjud emas, chunki, x→∞ da, kasrning surat hamda maxraji [0;2] kesmadagi har qanday qiymatini qabul qilish mumkin, hosilalarning nisbati esa, ixtiyoriy manfiy bo‘lmagan qiymatlarni qabul qiladi. Demak, bu holatda Lopital qoidasini qo‘llash mumkin emas.

2-misol. $\lim\limits_{x\to 0}\frac{e^{3x}-1}{\sin 5x}$ ni hisoblansin.

Yechish: Bu yerda Lopital qoidasining barcha shartlari bajarilmoqda. Shuning uchun,

$$\lim_{x\to 0}\frac{e^{3x}-1}{\sin 5x}=\lim_{x\to 0}\frac{3e^{3x}}{5\cos 5x}=\frac{3}{5}.$$

Agar 0·∞ yoki kabi aniqmaslikni ochish lozim bo‘lsa, uni yoki $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ga keltirib, so‘ngra Lopital qoidasi qo‘llaniladi.

3-misol. $\lim\limits_{x\to\infty}x^3e^{-x}$ ni hisoblang.

Yechish: Bu yerda 0·∞ kabi aniqmaslikni ochishga to‘g‘ri keladi, uni quyidagicha o‘zgartiramiz:

$$\lim_{x\to\infty}x^3e^{-x}=\lim_{x\to\infty}\frac{x^3}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{3x^2}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{6x}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{6}{e^x}=\frac{6}{e^\infty}=0.$$

Agar $\lim\limits_{x\to x_0}f_1(x)=\infty$ va $\lim\limits_{x\to x_0}f_2(x)=\infty$ bo‘lsa, u holda $f_1(x)$-$f_2(x)$ ning limiti ∞-∞ aniqmaslikka olib keladi. Ammo, $f_1(x)-f_2(x)=f_1(x)\left(1-\frac{f_2(x)}{f_1(x)}\right)$ deb yozsak, hamda $\lim\limits_{x\to x_0}\frac{f_2(x)}{f_1(x)}=1$ bo‘lsa, 0·∞ kabi aniqmaslikka kelamiz.

Endi $f(x)^{\varphi(x)}$ kabi funksiyani qaraymiz.

1. Agar $\lim\limits_{x\to x_0}f(x)=0$, $\lim\limits_{x\to x_0}\varphi(x)=0$ bo‘lsa, 0^0 aniqmaslik hosil bo‘ladi.

2. Agar $\lim\limits_{x\to x_0}f(x)=1$, $\lim\limits_{x\to x_0}\varphi(x)=0$ bo‘lsa, 1^∞ aniqmaslikka ega bo‘lamiz.

3. Agar $\lim\limits_{x\to x_0} f(x)=\infty$, $\lim\limits_{x\to x_0}\varphi(x)=0$ bo‘lsa, ∞^0 aniqmaslik hosil bo‘ladi.

Bu xildagi aniqmasliklarni ochib chiqish uchun logarifmlash usuli deb ataluvchi usuldan foydalanamiz, ya’ni faraz qilaylik, $\lim\limits_{x\to x_0} f(x)^{\varphi(x)}=A$ bo‘lsin. Logarifimik funksiyaning uzluksiz ekanligidan, $\lim\limits_{x\to x_0}\ln y=\ln\left(\lim\limits_{x\to x_0} y\right)$ ni yozish mumkin. U holda, $\ln A=\lim\limits_{x\to x_0}(\varphi(x)\cdot\ln f(x))$. Natijada, yuqoridagi aniqmasliklarning barchasi $0\cdot\infty$ ga keltiriladi.

4-misol. $\lim\limits_{x\to 0}(e^x+x)^{\frac{1}{x}}$ ni hisoblansin.

Yechish: Hisoblanishi lozim bo‘lgan limitni A deb belgilaymiz. U holda:

$$\ln A=\lim_{x\to 0}\left(\frac{1}{x}\cdot\ln(e^x+x)\right)=\lim_{x\to 0}\frac{\ln(e^x+x)}{x}=\lim_{x\to 0}\frac{e^x+1}{e^x+x}=2.$$ Demak $A=e^2$.

5.6. HOSILANING FUNKSIYA VA UNING GRAFIGINI TEKSHIRISHGA QO‘LLANILISHI

Differensial hisobning muhim amaliy masalalaridan biri funksiya o‘zgarishini tekshirishning umumiy usullarini ishlab chiqishdir.

Biror X oraliqda aniqlangan $y=f(x)$ funksiya argument x ning shu oraliqdagi $x_1<x_2$ shartni qanoatlantiruvchi qiymatlari uchun $f(x_1)<f(x_2)$ (yoki $f(x_1)>f(x_2)$) tengsizlikni qanoatlantirsa, u holda, funksiyani shu oraliqda o‘suvchi (yoki kamayuvchi) deb ataladi.

Funksiyaning o‘sishi (*kamayishi*) belgilarini sanab o‘tamiz:

1. Agar $[a;b]$ kesmada differensiallanuvchi bo‘lgan $y=f(x)$ funksiya o‘suvchi (yoki kamayuvchi) bo‘lsa, u holda kesmaning barcha nuqtalarida funksiyaning hosilasi manfiy (yoki musbat) bo‘la olmaydi, yani, $f'(x)\geq 0$ (yoki $f'(x)\leq 0$) dir.

2. Agar $[a;b]$ kesmada uzluksiz hamda kesmaning ichida differensiallanuvchi bo‘lgan funksiya musbat (yoki manfiy) hosilaga ega bo‘lsa, u holda funksiya shu kesmada o‘suvchi (yoki kamayuvchi) bo‘ladi.

Shuningdek, agar $y=f(x)$ funksiya biror X oraliqning ixtiyoriy $x_1<x_2$ qiymatlarida $f(x_1)\leq f(x_2)$ (yoki $f(x_1)\geq f(x_2)$) kabi shartni qanoatlantirsa, uni shu oraliqda *kamaymaydigan* (yoki *o‘smaydigan*) funksiya deyiladi.

Funksiyaning kamaymaydigan yoki o‘smaydigan oraliqlarini uning *monotonlik oraliqlari* deb yuritiladi. Funksiyaning aniqlanish sohasidagi monotonligining xarakteri, faqatgina uning birinchi tartibli hosilasining ishorasi o‘zgaradigan nuqtalardagina o‘zgarishi mumkin. Funksiyaning birinchi tartibli hosilasi nolga yoki uzilishga ega bo‘ladigan nuqtalar, uning *kritik nuqtalari* deb ataladi.

1-misol. $y=2x^2-lnx$ funksiyaning kritik nuqtalari va monotonlik oraliqlari aniqlansin.

Yechish: Funksiya $x>0$ qiymatlarda aniqlangan. Uning hosilasi, $y'=4x-\frac{1}{x}=\frac{4x^2-1}{x}$ bo‘lib, $x_0=\frac{1}{2}$ da $y'=0$ bo‘lganligi uchun kritik nuqta orqali funksiyaning aniqlanish sohasini $\left(0;\frac{1}{2}\right)$ va $\left(\frac{1}{2};+\infty\right)$ larga ajratamiz. Birinchi oraliqda $y'<0$ bo‘lib, ikkinchi oraliqda esa, $y'>0$. Demak, funksiya $(0;0,5)$ da kamayuvchi bo‘lib, $(0,5;+\infty)$ da esa o‘suvchi bo‘lar ekan.

Tarifga binoan, $y=f(x)$ funksiya uchun har qanday yetarlicha kichik $|\Delta x|\neq 0$ larda $f(x_1+\Delta x)<f(x_1)$ kabi tengsizlik o‘rinli bo‘lsa, u holda x_1 nuqtani funksiyaning *lokal maksimumi* deb ataladi. Aksincha, har qanday yetarlicha kichik $|\Delta x|\neq 0$ lar uchun $f(x_2+\Delta x)>f(x_2)$ shart bajarilsa, x_2 ni funksiyaning *lokal minimumi* deb ataladi. Maksimum va minimum nuqtalar birgalikda funksiyaning *ekstremum nuqtalari* deyilib, funksiyaning maksimumi bilan minimumi birgalikda uning *ekstremal qiymatlari* deb ataladi.

1-teorema (***lokal ekstremum mavjudligining zaruriy belgisi***). *Agar* $x = x_0$ *nuqtada* $y=f(x)$ *funksiya ekstremumga yerishadigan bo'lsa, u holda, yoki* $f'(x_0)=0$ *bo'ladi, yoki* $f'(x_0)$ *mavjud bo'lmaydi.*

Differensiallanuvchi funksiyaning ekstremum nuqtalarida uning grafigiga o'tkazilgan *urinmalar* har doim *Ox* o'qiga parallel bo'ladi.

2-misol. $y=(x+1)^3$ funksiyani ekstremumga tekshirilsin.

Yechish: Funksiyaning hosilasi $u'=3(x+1)^2$ bo'lib, $x=-1$ da $u'=0$ bo'ladi. Ammo, $x=-1$ da funksiya ekstremumga ega emas, chunki, $x>-1$ da $(x+1)^3>0$ bo'lib, $x<-1$ da esa, $(x+1)^3<0$ dir, hamda $x=-1$ da $(x+1)^3=0$. Demak, funksiya hosilasining nolga aylanishi, uning ekstremumi mavjudligini taminlamas ekan.

3-misol. $u=|x|$ funksiyani ekstremumga tekshirilsin.

Yechish: $y(0)=0$ bo'lganligidan, hamda $x\neq 0$ da $y=|x|>0$ ligi uchun $x=0$, minimum nuqtadir. Lekin, §6.1 ning 2-misolida funksiyaning $x=0$ nuqtada hosilasi mavjud emasligi ko'rsatilgan edi.

Yuqoridagi misollardan ko'rinadiki, har qanday kritik nuqtalarda ham funksiya ekstremumga ega bo'lavermas ekan. Ammo, biror nuqtada funksiya ekstremumga yerishadigan bo'lsa, u nuqta har doim ham funksiyaning kritik nuqtasi bo'ladi.

Funksiyaning ekstremumini topish uchun quyidagicha ish yuritiladi: barcha kritik nuqtalar aniqlangandan so'ng, ularning har birida alohida-alohida funksiyaning ekstremumi mavjudligi yoki umuman ekstremum mavjud emasligi tekshiriladi.

2-teorema (***lokal ekstremum mavjudlining birinchi yetarli sharti***) *Aytaylik, u=f(x) funksiya, biror $x=x_0$ kritik nuqta yotadigan oraliqda uzluksiz bo'lib, u oraliqning barcha nuqtalarida differensiallanuvchi (x_0 nuqtada differensiallanuvchi bo'lmasligi ham mumkin) bo'lsin. Agar $x<x_0$ da $f'(x)>0$, $x>x_0$ da esa, $f'(x)<0$ bo'ladigan bo'lsa, funksiya $x=x_0$ da maksimumga erishadi. Aksincha, yani, $x<x_0$ da $f'(x)<0$ va $x>x_0$ da $f'(x)>0$ bo'lsa, funksiya $x=x_0$ nuqtada minimumga yerishadi.*

Teorema shartida keltirilgan tengsizliklar, $x=x_0$ kritik nuqtaning yetarlicha kichik atrofida bajarilishi lozimligini yana bir bor takidlaymiz. Birinchi tartibli hosila yordamida funksiyani ekstremumga tekshirish sxemasi jadval ko'rinishida yozilishi mumkin (6.1-jadvalga qaralsin).

4-misol. $y=2x+3\sqrt[3]{x^2}$ funksiyani ekstremumga tekshirilsin.

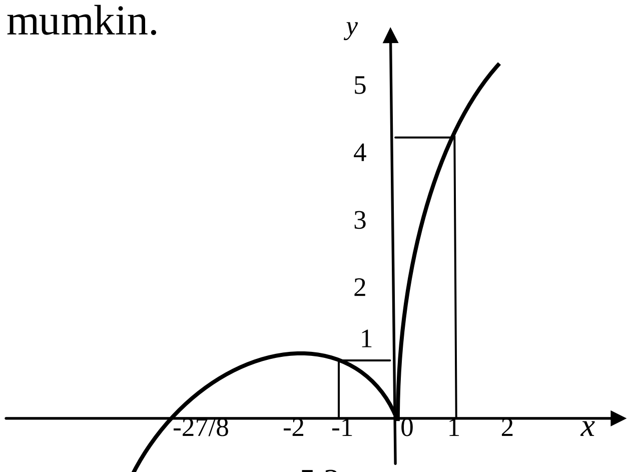
Yechish: Qaralayotgan funksiya barcha $x\in R$ lar uchun aniqlangan hamda uzluksiz. $y'=2+\frac{2}{\sqrt[3]{x}}=\frac{2}{\sqrt[3]{x}}\left(\sqrt[3]{x}+1\right)$ ekanligi uchun, kritik nuqtalar $x_1=-1$ bilan $x_2=0$ lardir, chunki ularning birinchisida y'=0 bo'lib, ikkinchisida esa, y' uzilishga ega. Bu nuqtalar funksiyaning aniqlanish sohasini (-∞;-1), (-1;0) va (0;+∞) kabi oraliqlarga ajratadi. Bu oraliqlarda hosila o'z ishorasini saqlaydi. (∞; -1) da, y'>0, ya'ni, funksiya o'suvchi; (-1;0) da esa, u'<0 va funksiya kamayuvchi; (0;+∞) da, $y'>0$ va funksiya o'suvchidir. Demak, $x_1=-1$ kritik nuqta funksiyaning lokal maksimum nuqtasi bo'lib, $y_{max}=y(-1)=1$; $x_2=0$ nuqta esa, lokal minimum nuqta bo'lib, $y_{min}=y(0)=0$. (5.1-rasmga qaralsin).

5.1- jadval

$f'(x)$ ning x_0 kritik nuqta atrofidagi ishoralari			Kritik nuqtaning xaraktyeri
$x<x_0$	$x=x_0$	$x>x_0$	
+	$f'(x_0)=0$ yoki mavjud emas	-	Maksimum nuqtasi
-	— I I —	+	Minimum nuqtasi
+	— I I —	+	Ekstremum mavjud emas (f-ya o‘suvchi)
-	— I I —	-	Ekstremum mavjud emas (f-ya kamayuvchi)

3-teorema ***(lokal ekstremum mavjudligining ikkinchi yetarli sharti).*** *Aytaylik, $y=f(x)$ funksiya, biror $x_0 \in X$ oraliqda ikki marta differensiallanuvchi bo‘lib, $f'(x_0)=0$ bo‘lsin. U holda, agar, $f''(x_0)<0$ shart bajarilsa, funksiya $x=x_0$ nuqtada lokal maksimumga erishadi, aksincha, $f''(x_0)>0$ bo‘lsa, lokal minimumga erishadi.*

Agarda, $f''(x_0)=0$ bo‘lsa, $x=x_0$ nuqtada ekstremum mavjud bo‘lmasligi ham mumkin.



5.3-rasm.

5- misol. $y=x^2e^{-x}$ funksiyani ikkinchi tartibli hosila yordamida ekstremumga tekshirilsin.

Yechish: Birinchi hamda ikkinchi tartibli hosilalarni hisoblaymiz.

$y'=2xe^{-x}-x^2e^{-x}=(2x-x^2)e^{-x}$, $y''=(2-2x)e^{-x}-(2x-x^2)e^{-x}=(x^2-4x+2)e^{-x}$,

$y'=0$ dan, $x_1=0$ va $x_2=2$ kritik nuqtalarni aniqlaymiz. Ikkinchi tartibli hosilaning kritik nuqtalaridagi qiymatlari, $y''(0)=2>0$ va $y''(2)=-2e^{-2}<0$ bo‘lganliklaridan, $x_1=0$ nuqtada funksiya minimumga, $x_2=2$ nuqtada esa, maksimumga ega bo‘ladi; $y_{min}=y(0)=0$, $y_{max}=y(2)=4e^{-2}$.

Qoidaga binoan, $[a;b]$ kesmada uzluksiz bo‘lgan $y=f(x)$ funksiya o‘zining eng kichik m va eng katta M qiymatlariga yoki $(a;b)$ ochiq oraliqda yotuvchi kritik nuqtalarda yoki kesmaning chegaralarida erishishi mumkin.

6-misol. $y=x^3-3x+3$ funksiyaning [-2;3] kesmadagi eng kichik va eng katta qiymatlari topilsin.

Yechish: Funksiyaning hosilasi $y'=3x^2-3$ ekanligidan, $x_1=-1$ bilan $x_2=1$ lar funksiyaning kritik nuqtalaridir va ular (-2;3) oraliqda yotadi. Shuning uchun: $y(-1)=5$, $y(1)=1$, $y(-2)=1$ va $y(3)=21$ larni aniqlaymiz. Ushbu aniqlangan qiymatlarni o‘zaro taqqoslab, funksiyaning eng kichik qiymati $m=1$ ga va eng katta qiymati $M=21$ ga teng ekanliklarini topamiz.

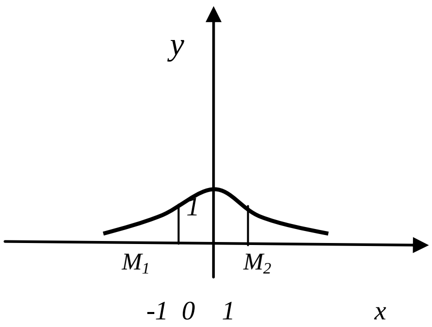
Aytaylik, egri chiziq $(a;b)$ oraliqda o‘zining $u=f(x)$ funksiya bilan ifodalanadigan tenglamasi bilan berilgan bo‘lsin. U holda,egri chiziqning barcha nuqtalari uning har qanday nuqtasiga o‘tkazilgan urinmadan pastda joylashgan bo‘lsa, uni $(a;b)$ oraliqda *qavariq* deb aytiladi. Aksincha bo‘lsa, ya’ni, yuqorida joylashsa, uni $(a;b)$ oraliqda *botiq* deyiladi. Egri chiziqning biror $M_0(x_0; f(x_0))$ nuqtasi, uning qavariqlik qismini botiqlik qismidan ajratgan bo‘lsa, uni egri chiziqning *burilish nuqtasi* deb yuritiladi. Bu holda M_0 nuqtada o‘tkazilgan urinma mavjud deb faraz qilinadi.

4-teorema (***funksiya grafigining qavariqligi (botiqligi) ning yetarli sharti***) *Agar $(a;b)$ oraliqning barcha nuqtalarida $y=f(x)$ funksiyaning ikkinchi tartibli hosilasi manfiy (musbat) bo‘lsa, ya’ni, $f''(x)<0$ $(f''(x)>0)$*

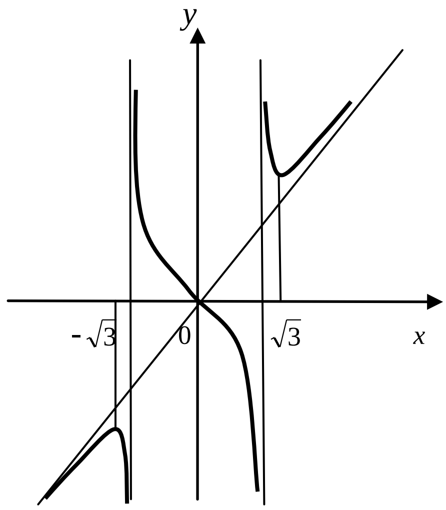
bo'lsa, u holda y=f(x) egri chiziq (a;b) oraliqda har doim qavariq (botiq) dir.

Egri chiziqning burilish nuqtasining chap va o'ng tomonida *u''* hosilaning ishoralari har xil bo'lganligi bois, bu nuqtada $y''=0$ yoki $y''=\infty$ dir.

5-teorema (***burilish nuqtasi mavjudligining yetarli sharti***). *Agar $x=x_0$ nuqtada $f''(x_0)=0$ yoki $f''(x_0)$ mavjud bo'lmasa, hamda $f''(x)$ hosila $x=x_0$ nuqtadan o'tganda ishorasini o'zgartiradigan bo'lsa, u holda abssissasi $x=x_0$ bo'lgan nuqta $y=f(x)$ egri chiziqning burilish nuqtasidir*.



5.4- rasm



5.5-rasm.

7-Misol . $y=\dfrac{x^3}{x^2-1}$ egri chiziqning asimptotalari topilsin.

Yechish: $\lim\limits_{x\to\pm1}\dfrac{x^3}{x^2-1}=\pm\infty$ bo'lganligidan, egri chiziq ikkita $x=\pm1$ vertikal asimptotalarga ega. Og'ma asimptotalarni qidiramiz.

$$k=\lim_{x\to\pm\infty}\frac{y}{x}=\lim_{x\to\pm\infty}\frac{x^2}{x^2-1}=1 \qquad b=\lim_{x\to\pm\infty}(f(x)-kx)=\lim_{x\to\pm\infty}\left(\frac{x^3}{x^2-1}-1\right)=\lim_{x\to\pm\infty}\frac{x}{x^2-1}=0$$

Demak, qaralayotgan egri chiziqning bitta $u=x$ og'ma asimptotasi mavjud ekan.(6.5-rasm).

5.7. FUNKSIYANI TO'LA TEKSHIRISH VA UNING GRAFIGINI YASASH SXEMASI

Funksiyani to'la tekshirish va uning grafigini yasash uchun quyidagi sxemani tavsiya etish mumkin:

1) funksiyaning aniqlanish sohasini ko'rsatish;

2) funksiyaning uzilish nuqtalari, uning grafigining koordinata o'qlari bilan kesishish nuqtalari hamda vertikal asimptotalari (agar ular mavjud bo'lsalar)ni aniqlash;

3) funksiyaning juft yoki toqligi, davriyligini aniqlash;

4) funksiyani monotonlikka va ekstremumga tekshirish;

5) qavariqlik, botiqlik oraliqlari hamda burilish nuqtalarini aniqlash;

6) funksiya grafigining asimptotalarini topish;

7) boshqa kerakli qo'shimcha hisoblashlarni bajarish;

8) funksiyaning grafigini yasash.

Misol $y=\sqrt[3]{(x+3)x^2}$ funksiyani to'la tekshirib uning grafigi yasalsin.

Yechish: 1. Funksiya barcha $x \in R$ lar uchun aniqlangan.

2. Funksiyaning uzilish nuqtalari mavjud emas, grafik *Ox* o'qini *x*=-3 va *x*=0 nuqtalarda kesib, koordinata boshidan o'tadi.

3. Funksiya juft ham, toq ham, davriy ham emas.

4. Hosilani hisoblaymiz: $f'(x)=\dfrac{x+2}{\sqrt[3]{x(x+3)^2}}$.

Hosila, x_1=-2 da 0 ga teng va x_2=-3 bilan x_3=0 nuqtalarda mavjud emas. Ushbu nuqtalar, funksiyaning aniqlanish sohasini (-∞:-3), (-3;-2), (-2;0) va (0;+∞) kabi oraliqlarga ajratadi. Ulardan, (-∞:-3), (-3;-2) va (0;+∞) oraliqlarda $f'(x)>0$ bo'lib, (-2;0) oraliqda esa, $f'(x)<0$ dir. Bundan ko'rinmoqdaki, funksiya (-∞:-2) va (0;+∞) oraliqlarda o'suvchi bo'lib, (-

2;0) oraliqda esa, kamayuvchidir. Shuningdek x_1=-2 maksimum nuqtasi bo‘lib, $y_{\max} = \sqrt[3]{4}$ hamda x_3=0 esa, minimum nuqtasidir va $y_{min}=y(0)=0$; x_2 =-3 nuqtada funksiya ekstremumga ega emas.

5. Ikkinchi tartibli hosilani topamiz: $f''(x) = -\dfrac{2}{\sqrt[3]{x^4(x+3)^5}}$

bu hosila argument x ning hech bir chekli qiymatida 0 ga teng bo‘la olmaydi. Shu sababli, ikkinchi tartibli hosila mavjud bo‘lmaydigan nuqtalargina ya’ni, x_2=-3 bilan x_3=0 nuqtalargina egri chiziq burilish nuqtalarining absissalari bo‘lishi mumkin. Ushbu nuqtalar orqali, funksiyaning aniqlanish sohasini (-∞:-3), (-3;0) va (0;+∞) kabi bo‘laklarga ajratib, ularning har birida $f(x)$ ning ishoralarini aniqlaymiz: (-∞:-3) oraliqda $f''(x)>0$ bo‘lganligi uchun u oraliqda egri chiziq botiq bo‘lib, (-3;0) bilan (0;+∞) oraliqlarda $f''(x)<0$ bo‘lganligidan, egri chiziq u oraliqlarda qavariqdir. x_2=-3 nuqtaning atrofida $f''(x)$ ning ishoralari turli xil bo‘lganligi uchun M(-3;0) nuqta, egri chiziqning burilish nuqtasidir. Ammo, x_3=0 nuqta atrofida $f''(x)$ ning ishorasi bir xil bo‘lganligi sababli, u burilish nuqtasi bo‘la olmaydi.

6. Qaralayotgan funksiya cheksiz uzilish nuqtalariga ega bo‘lmaganligi bois, vertikal asimptotalari yo‘q. Og‘ma asimptotalar mavjudligini tekshiramiz:

$$k = \lim_{x\to\pm\infty} \frac{y}{x} = \lim_{x\to\pm\infty} \frac{\sqrt[3]{(x+3)x^2}}{x} = \lim_{x\to\pm\infty} \sqrt[3]{1+\frac{3}{x}} = 1,$$

$$b=\lim_{x\to\pm\infty}(y-kx)=\lim_{x\to\pm\infty}\left(\sqrt[3]{(x+3)x^2}-x\right)=\lim_{x\to\pm\infty}\frac{\left(\sqrt[3]{(x+3)x^2}-x\right)\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}{\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}=$$

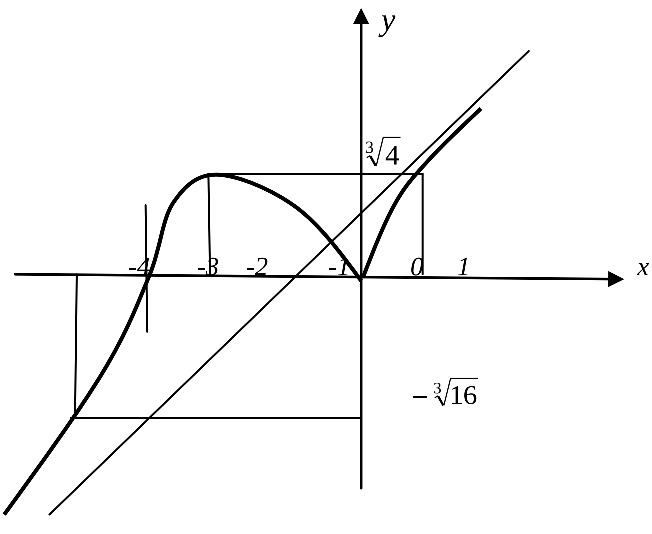
$$=\lim_{x\to\pm\infty}\frac{(x+3)x^2-x^3}{\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}=\lim_{x\to\pm\infty}\frac{3x^2}{\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}=$$

$$=\lim_{x\to\pm\infty}\frac{3}{\left(\sqrt[3]{(1+\frac{3}{x})^2}+\sqrt[3]{(1+\frac{3}{x})}+1\right)}=1,$$

Demak, $y=x+1$ to‘g‘ri chiziq og‘ma asimptota bo‘lar ekan.

7. Funksiyaning grafigini chizishdan avval, grafik Ox o‘qini $x_2=-3$ va $x_3=0$ nuqtalarda qanday α burchak ostida kesib o‘tishligini aniqlash maqsadga muvofiqdir. Bu nuqtalarda $u'=tg\alpha=\infty$ ligi uchun $\alpha=\frac{\pi}{2}$ ga teng bo‘ladi. Agar $x_3=0$ nuqta funksiyaning minimum nuqtasi bo‘lganligi sababli, funksiyaning grafigi bu nuqta atrofida Ox o‘qidan yuqorida joylashgan bo‘ladi, hamda $x_3=0$ nuqta funksiya grafigining *“qaytishi” nuqtasidir*.

8. Tekshirishlar natijasiga ko‘ra, qaralayotgan funksiyaning grafigini yasaymiz. (5.6-rasm).



5.6-rasm.

DIFFERENSIAL HISOB BO'LIMIGA TOPSHIRIQ VARIANTLARI

BERILGAN FUNKSIYALARNI DIFFERENSIYALLANG

1– TOPSHIRIQ

1. $y = 2x^5 - \frac{4}{x^3} + \frac{1}{x} + 3\sqrt{x}.$

2. $y = \frac{3}{x} + \sqrt[5]{x^2} - 4x^3 + \frac{2}{x^4}.$

3. $y = 3x^4 + \sqrt[3]{x^5} - \frac{2}{x} - \frac{4}{x^2}.$

4. $y = 7\sqrt{x} - \frac{2}{x^5} - 3x^3 + \frac{4}{x}.$

5. $y = 7x + \frac{5}{x^2} - \sqrt[7]{x^4} + \frac{6}{x}.$

6. $y = 5x^2 - \sqrt[3]{x^4} + \frac{4}{x^3} - \frac{5}{x}.$

7. $y = 3x^5 - \frac{3}{x} - \sqrt{x^3} + \frac{10}{x^5}.$

8. $y = \sqrt[3]{x^7} + \frac{3}{x} - 4x^6 + \frac{4}{x^5}.$

9. $y = 8x^2 + \sqrt[3]{x^4} - \frac{4}{x} - \frac{2}{x^3}.$

10. $y = 4x^6 + \frac{5}{x} - \sqrt[3]{x^7} - \frac{7}{x^4}.$

11. $y = 2\sqrt{x^3} - \frac{7}{x} + 3x^2 - \frac{2}{x^5}.$

12. $y = 4x^3 - \frac{3}{x} - \sqrt[5]{x^2} + \frac{6}{x^2}.$

13. $y = 5x^3 - \frac{8}{x^2} + 4\sqrt{x} + \frac{1}{x}.$

14. $y = \frac{9}{x^3} + \sqrt[3]{x^4} - \frac{2}{x} + 5x^4.$

15. $y = \frac{4}{x^5} - \frac{9}{x} + \sqrt[5]{x^2} - 7x^3.$

16. $y = \frac{8}{x^3} + \frac{3}{x} - 4\sqrt{x^3} + 2x^7.$

17. $y = 5x^2 - \frac{4}{x} - \sqrt[3]{x^7} - 2x^6.$

18. $y = 10x^2 + 3\sqrt{x^5} - \frac{4}{x} - \frac{5}{x^4}.$

19. $y = \sqrt{x^5} - \frac{3}{x} + \frac{4}{x^3} - 3x^2.$

20. $y = 9x^3 + \frac{5}{x} - \frac{7}{x^4} + \sqrt[3]{x^7}.$

21. $y = 3\sqrt{x} + \frac{4}{x^5} + \sqrt[3]{x^2} - \frac{7}{x}.$

22. $y = \sqrt{x^3} + \frac{2}{x} - \frac{4}{x^5} - 5x^3.$

23. $y = 7x^2 + \frac{3}{x} - \sqrt[5]{x^4} + \frac{8}{x^3}.$

24. $y = 8x^3 - \frac{4}{x} - \frac{7}{x^4} + \sqrt[7]{x^2}.$

25. $y = 8x - \frac{5}{x^4} + \frac{1}{x} - \sqrt[5]{x^4}.$

26. $y = \sqrt[4]{x^3} - \frac{5}{x} + \frac{4}{x^5} + 3x.$

27. $y = 4x^3 + \frac{3}{x} - \sqrt[3]{x^5} - \frac{2}{x^4}.$

28. $y = 4x^5 - \frac{5}{x} - \sqrt{x^3} + \frac{2}{x^3}.$

29. $y = \frac{7}{x} + \frac{4}{x^3} - \sqrt[5]{x^3} - 2x^6.$

30. $y = \frac{6}{x^4} - \frac{3}{x} + 3x^3 - \sqrt{x^7}.$

2– TOPSHIRIQ

1. $y=\sqrt[3]{3x^4+2x-5}+\dfrac{4}{(x-2)^5}$.

2. $y=\sqrt[3]{(x-3)^4}-\dfrac{3}{2x^3-3x+1}$.

3. $y=\sqrt{(x-4)^5}+\dfrac{5}{(2x^2+4x-1)^2}$.

4. $y=\sqrt[5]{7x^2-3x+5}-\dfrac{5}{(x-1)^3}$.

5. $y=\sqrt[4]{3x^2-x+5}-\dfrac{34}{(x-5)^4}$.

6. $y=\sqrt{3x^4-2x^3+x}-\dfrac{4}{(x+2)^3}$.

7. $y=\sqrt[3]{(x-7)^5}+\dfrac{5}{4x^2+3x-5}$.

8. $y=\sqrt[5]{(x+4)^6}-\dfrac{2}{2x^2-3x+7}$.

9. $y=\dfrac{3}{(x-4)^7}-\sqrt{5x^2-4x+3}.$

10. $y=\sqrt[3]{4x^2-3x-4}-\dfrac{2}{(x-3)^5}$.

11. $y=\dfrac{7}{(x-1)^3}+\sqrt{8x-3+x^2}$.

12. $y=\sqrt[3]{4x^2-3x-4}-\dfrac{2}{(x-3)^5}$.

13. $y=\sqrt[3]{5x^4-2x-1}+\dfrac{8}{(x-5)^2}$.

14. $y=\dfrac{3}{(x+2)^5}-\sqrt[7]{5x-7x^2-3}.$

15. $y=\sqrt[4]{(x-1)^5}-\dfrac{4}{7x^2-3x+2}$.

16. $y=\sqrt[5]{(x-2)^6}-\dfrac{3}{7x^3-x^2-4}$.

17. $y=\dfrac{3}{(x+4)^2}-\sqrt[3]{4+3x-x^4}$.

18. $y=\dfrac{2}{(x-1)^3}-\dfrac{8}{6x^2+3x-7}$.

19. $y=\sqrt{1+5x-2x^2}+\dfrac{3}{(x-3)^4}$.

20. $y=\sqrt[3]{5+4x-x^2}-\dfrac{5}{(x+1)^3}$.

21. $y=\sqrt[4]{5x^2-4x+1}-\dfrac{7}{(x-5)^2}$.

22. $y=\sqrt[5]{3-7x+x^2}-\dfrac{4}{(x-7)^5}$.

23. $y=\sqrt{(x-3)^7}+\dfrac{9}{7x^2-5x-8}$.

24. $y=\sqrt[3]{(x-8)^4}-\dfrac{2}{1+3x-4x^2}$.

25. $y=\dfrac{3}{4x-3x^2+1}-\sqrt{(x+1)^5}$.

26. $y=\dfrac{3}{x-4}+\sqrt[6]{(2x^2-3x+1)^5}$

27. $y=\dfrac{4}{(x-7)^3}-\sqrt[3]{(3x^2-x+1)^4}$.

28. $y=\sqrt{(x-4)^7}-\dfrac{10}{(3x^2-5x+1)}$.

29. $y=\dfrac{7}{(x+2)^5}-\sqrt{8-5x+2x^2}$.

30. $y=\sqrt[3]{(x-1)^5}+\dfrac{5}{2x^2-4x+7}$.

3– TOPSHIRIQ

1. $y=\sin^3 2x\cdot\cos 8x^5.$

2. $y=\cos^5 3x\cdot tg(4x+1)^3.$

3. $y=tg^4 x\cdot\arcsin 4x^5.$

4. $y=\arcsin^3 2x\cdot ctg 7x^4.$

5. $y=ctg 3x\ \cdot\arccos 3x^2.$

6. $y=\arccos^2 4x\cdot\ln(x-3).$

7. $y=\ln^5 x\cdot arctg 7x^4.$

8. $y=arctg^3 4x\cdot 3^{\sin x}.$

9. $y=2^{\cos x}\cdot arcctg 5x^3.$

10. $y=4^{-x}\cdot\ln^5(x+2).$

11. $y=3^{tgx}\cdot\arcsin 7x^4.$

12. $y=5^{x^2}\cdot\arccos 2x^5.$

13. $y=\sin^4 3x\cdot arctg 2x^3.$

14. $y=\cos^3 4x\cdot arcctg\sqrt{x}.$

15. $y=tg^3 2x\cdot\arcsin x^5.$

16. $y=ctg^7 x\cdot\arccos 2x^3.$

17. $y=e^{-\sin x} tg 7x^6$

18. $y=e^{\cos x} ctg 8x^3.$

19. $y=\cos^5 x\cdot\arccos 4x.$

20. $y=\sin^3 7x\cdot arcctg 5x^2.$

21. $y=\sin^2 3x\cdot arcctg 3x^5.$

22. $y=\cos\sqrt[5]{x}\cdot arctg x^4.$

23. $y=tg^6 2x\cdot\cos 7x^2.$

24. $y=ctg^3 4x\cdot\arcsin\sqrt{x}.$

25. $y=ctg\,\frac{1}{x}\cdot\arccos x^4.$

26. $y=tg\sqrt{x}\cdot arcctg 3x^5.$

27. $y=tg^3 2x\cdot\arccos 2x^3.$

28. $y=2^{tgx} arctg^5 3x.$

29. $y=\sin^5 3x\cdot arctg\sqrt{x}.$

30. $y=\cos^4 3x\cdot\arcsin 3x^2.$

4– TOPSHIRIQ

1. $y=arcctg^2 5x\cdot\ln(x-4).$

2. $y=arctg^3 2x\cdot\ln(x+5).$

3. $y=\arccos^4 x\cdot\ln(x^2+x-1).$

4. $y=\sqrt{\arccos 2x}\cdot 3^{-x}.$

5. $y=tg^4 3x\cdot arctg 7x^2.$

6. $y=5^{-x^2}\arcsin 3x^3.$

7. $y=arctg^5 x\cdot\log_2(x-3).$

8. $y=\log_3(x+5)\cdot\arccos 3x.$

9. $y=e^{-x}\cdot\arcsin^2 5x.$

10. $y=\log_4(x-1)\cdot\arcsin^4 x.$

11. $y=(x-4)^5\cdot arcctg 3x^2.$

12. $y=ctg^3 4x\cdot arctg 2x^3.$

13. $y=e^{-\cos x}\cdot arctg 7x^5.$

14. $y=(x+1)\arccos 3x^4.$

15. $y=2^{\sin x} arcctg x^4.$

16. $y=3^{-x^3} arctg 2x^5.$

17. $y=3^{\cos x}\arcsin^2 3x.$

18. $y=\ln(x-10)\cdot arccox^2 4x.$

19. $y=\lg(x-2)\cdot\arcsin^5 x.$

20. $y=\log_3(x+1)\cdot arctg^5 7x.$

21. $y=\ln(x+9)\cdot arcctg^3 2x.$

22. $y=\lg(x+2)\cdot\arcsin^2 3x.$

23. $y=4^{-\sin x} arctg 3x.$

24. $y=2^{\cos x} arcctg^3 x.$

25. $y=\lg(x-3)\cdot\arcsin^2 5x.$

26. $y=\log_2(x+3)\cdot\arccos^2 x.$

27. $y=2^{-x} arctg^3 4x.$

28. $y=\ln(x-4)\cdot arcctg^4 3x.$

29. $y=\lg(x+3)\cdot arcctg^2 5x.$

30. $y=\log_5(x+1)\cdot arctg^2 x^3.$

5– TOPSHIRIQ

1. $y=tg^4 3x\cdot\arcsin 2x^3.$

2. $y=(x-2)^4\arcsin 5x^4.$

3. $y=2^{-x^3} arctg 7x^4.$

4. $y=(x+6)^5 arcctg 3x^5.$

5. $y=3^{\cos x}\ln(x^2-3x+7).$

6. $y=\log_2(x-7)\cdot arctg\sqrt{x}.$

7. $y=\arccos^3 5x\cdot tgx^4.$

8. $y=(x-5)^7 arcctg 7x^3.$

9. $y=\arccos x^2\cdot ctg 7x^3.$

10. $y=5^{-x^2}\arccos 5x^4.$

11. $y=arctg^4 x\cdot\cos 7x^4.$

12. $y=4(x-7)^6\arcsin 3x^5.$

13. $y=(x+5)^4\arccos^3 5x.$

14. $y=2^{-\sin x}\arcsin^3 2x.$

15. $y=(x+2)^7\arccos\sqrt{x}.$

16. $y=(x-7)^5\arcsin 7x^4.$

17. $y=\ln(x-3)\cdot\arccos 3x^4.$

18. $y=\log_2(x-4)\cdot arctg^3 4x.$

19. $y=(x-4)^4 arcctg^2 7x.$

20. $y=\sqrt[3]{x-3}\arccos^4 2x.$

21. $y=\sqrt[3]{x-4}\arcsin^4 5x.$

22. $y=(x-3)^5\arccos 3x^6.$

23. $y=\sqrt{(x+3)^5}\arcsin 2x^3.$

24. $y=\sqrt[3]{(x+1)^2}\arccos 3x.$

25. $y=tg^3 x\cdot arcctg 3x.$

26. $y=\sqrt{(x-2)^3} arctg(7x-1).$

27. $y=\sqrt[5]{(x+4)^2}\arcsin 7x^2.$

28. $y=\arcsin^3 4x\cdot ctg 3x.$

29. $y=e^{-\cos x}\arcsin 2x.$

30. $y=\sqrt{(x+5)^3}\arccos^4 x.$

6– TOPSHIRIQ

1. $y=(x-3)^4\arccos 5x^3.$

2. $y=(3x-4)^4\arccos 3x^2.$

3. $y=sh^3 4x\cdot\arccos\sqrt{x}.$

4. $y=th^2\sqrt{x}\cdot arcctg\,3x^2.$

5. $y=cth^3 5x\cdot\arcsin 3x^2.$

6. $y=ch\frac{1}{x}\cdot arctg(7x+2).$

7. $y=ch^3 4x\cdot\arccos 4x^2.$

8. $y=sh^3 3x\cdot arcctg\,5x^2.$

9. $y=th^5 3x\cdot\arcsin\sqrt{x}.$

10. $y=cth^2(x+1)\cdot\arccos\frac{1}{x}.$

11. $y=sh^4 2x\cdot\arccos x^2.$

12. $y=ch^3(3x+2)\cdot arctg\,3x.$

13.6 $y=th^3 4x\cdot arcctg\,3x^4.$

14. $y=cth^4 7x\cdot\arcsin\sqrt{x}.$

15. $y=sh^3 2x\cdot\arcsin 7x^2.$

16. $y=th^5 4x\cdot\arccos 3x^4.$

17. $y=ch^2 5x\cdot arctg\sqrt{x}.$

18. $y=cth^4 2x\cdot arctgx^3.$

19. $y=sh^4 5x\cdot\arccos 3x^2.$

20. $y=ch^3 9x\cdot arctg(5x-1).$

21. $y=th^4 x\cdot arcctg\,\frac{1}{x}.$

22. $y=cth^3 4x\cdot\arcsin(3x+1).$

23. $y=ch^2 5x\cdot arctgx^4.$

24. $y=th^4 7x\cdot\arccos x^3.$

25. $y=cth 4x^5*\arccos 2x$.

26. $y=cth 3x*\arcsin^4 2x.$

27. $y=th^5 3x*arctg\sqrt{x}.$

28. $y=sh^4 3x*\arccos 5x^4$.

29. $y=cth^2 4x*\arcsin x^3.$

30. $y=th^3 5x*arctg(2x-5).$

7– TOPSHIRIQ

1. $y=\frac{e^{\arccos^3 x}}{\sqrt{x+5}}.$

2. $y=\frac{(x-4)^2}{e^{arcctgx}}.$

3. $y=\frac{e^{-x^3}}{\sqrt{x^2+5x-1}}.$

4. $y=\frac{e^{-ctg5x}}{(3x^2-4x+2)}.$

5. $y=\frac{\sqrt{7x^3-5x+2}}{e^{\cos x}}.$

6. $y=\frac{e^{tg3x}}{\sqrt{3x^2-x+4}}.$

7. $y=\frac{e^{\sin x}}{(x-5)^7}.$

8. $y=\frac{\sqrt[3]{2x^2-3x+1}}{e^{-x}}.$

9. 7.9 $y=\frac{\sqrt{x^3+4x-5}}{e^{x^3}}$

10. $y=\frac{e^{ctg5x}}{(x+4)^3}.$

11. $y=\frac{\sqrt{3+2x-x^2}}{e^x}$.

12. $y=\frac{e^{3x}}{\sqrt{3x^2-4x-7}}$.

13. $y=\frac{e^{-\sin 2x}}{(x+5)^4}$.

14. $y=\frac{e^{\cos 5x}}{\sqrt{x^2-5x-2}}$.

15. $y=\frac{(2x+5)^3}{e^{tgx}}$.

16. $y=\frac{e^{-tg3x}}{4x^2-3x+5}$.

17. $y=\frac{e^{-\sin 4x}}{(2x-5)^6}$.

18. $y=\frac{3x^2-5x+10}{e^{-x^4}}$.

19. $y=\frac{e^{-x}}{(2x^2-x+4)^2}$.

20. $y=\frac{e^{4x}}{(3x+5)^3}$.

21. $y=\frac{e^{ctg5x}}{(3x-5)^4}$.

22. $y=\frac{(2x-3)^7}{e^{-2x}}$.

23. $y=\frac{(3x+1)^4}{e^{4x}}$.

24. $y=\frac{5x^2+4x-2}{e^{-x}}$

25. $y=\frac{\sqrt{5x^2-x+1}}{e^{3x}}$.

26. $y=\frac{e^{-x^2}}{(2x-5)^7}$.

27. $y=\frac{e^{\cos 3x}}{(2x+4)^5}$.

28. $y=\frac{e^{\sin 5x}}{(3x-2)^2}$.

29. $y=\frac{\sqrt{x^2-3x-7}}{e^{-x^3}}$.

30. $y=\frac{e^{-tgx}}{4x^2+7x-5}$.

8– TOPSHIRIQ

1. $y=\frac{\log_5(3x-7)}{ctg7x^3}$.

2. $y=\frac{\ln(5x-3)}{4tg3x^4}$.

3. $y=\frac{\ln(7x+2)}{5\cos 42x}$.

4. $y=\frac{\sin^3 5x}{\ln(2x-3)}$.

5. $y=\frac{\cos^2 3x}{\lg(3x-4)}$.

6. $y=\frac{tg^3 2x}{\lg(5x+1)}$.

7. $y=\frac{\log_3(4x+5)}{2ctg\sqrt{x}}$.

8. $y=\frac{\ln(7x-3)}{3tg^2 4x}$.

9. $y=\frac{\lg(11x+3)}{\cos^2 5x}$.

10. $y=\frac{ctg^2 5x}{\ln(7x-2)}$.

11. $y=\frac{tg^2(x-2)}{\lg(x+3)}$.

12. $y=\frac{\sin^3(5x+1)}{\lg(3x-2)}$.

13. $y=\dfrac{\cos^4(7x-1)}{\lg(x+5)}.$

14. $y=\dfrac{\sin^3(4x+3)}{\ln(7x+1)}.$

15. $y=\dfrac{ctg^3(2x-3)}{\log_3(x+2)}.$

16. $y=\dfrac{\lg^3 x}{\sin 5x^2}.$

17. $y=\dfrac{\ln^2(x+1)}{\cos 3x^4}.$

18. $y=\dfrac{\log_2(7x-5)}{tg\sqrt{x}}.$

19. $y=\dfrac{\log_3(4x-2)}{ctg\,2x}.$

20. $y=\dfrac{\ln^3(x-5)}{tg(\frac{1}{x})}.$

21. $y=\dfrac{\lg(x+2)}{\sin 2x^5}.$

22. $y=\dfrac{tg^3 7x}{\ln(3x+2)}.$

23. $y=\dfrac{ctg\sqrt{x-2}}{\lg(3x+5)}.$

24. $y=\dfrac{tg(3x-5)}{\ln^2(x+3)}.$

25. $y=\dfrac{\cos^2 x}{\lg(x^2-2x+1)}.$

26. $y=\dfrac{\log_2(3x+7)}{tg3x}.$

27. $y=\dfrac{\ln^3 x}{ctg(x-3)}.$

28. $y=\dfrac{tg^4 5x}{\ln(x+7)}.$

29. $y=\dfrac{\log_3(x+4)}{\cos^5 x}.$

30. $y=\dfrac{tg^4 3x}{\lg(x^2-x+4)}.$

9– TOPSHIRIQ

1. $y=\dfrac{arcctg^4 5x}{sh\sqrt{x}}.$

2. $y=\dfrac{arctg^3 2x}{ch(\frac{1}{x})}.$

3. $y=\dfrac{ar\cos 3x^4}{th^2 x}.$

4. $y=\dfrac{\arcsin 5x^3}{ch\sqrt{x}}.$

5. $y=\dfrac{cth^3(x+1)}{\arccos 2x}.$

6. $y=\dfrac{th3x^5}{arctg^2 3x}.$

7. $y=\dfrac{\arccos^7 2x}{thx^5}.$

8. $y=\dfrac{\arcsin^3 4x}{sh(3x+1)}.$

9. $y=\dfrac{th^4(2x+5)}{\arccos 3x}.$

10. $y=\dfrac{\sqrt[3]{arctg\,2x}}{sh^2 x}.$

11. $y=\dfrac{\arcsin^2 4x}{th(5x-3)}.$

12. $y=\dfrac{ch^2(4x+2)}{arctgx^3}.$

13. $y=\dfrac{\arcsin 4x^5}{th^3 x}.$

14. $y=\dfrac{arctg^3(2x+1)}{ch\sqrt{x}}.$

15. $y=\dfrac{\arccos 4x^3}{sh^4x}.$

16. $y=\dfrac{cth^2(x-2)}{\arccos 3x}.$

17. $y=\dfrac{th^3(2x+2)}{\arcsin 5x}.$

18. $y=\dfrac{cth^2(3x-1)}{\arccos x^2}.$

19. $y=\dfrac{sh^5x}{\arccos 4x}.$

20. $y=\dfrac{\sqrt{ch^3x}}{arctg5x}.$

21. $y=\dfrac{th^2(x+3)}{arcctg\sqrt{x}}.$

22. $y=\dfrac{\arcsin^2 3x}{ch(x-5)}.$

23. $y=\dfrac{arcctg^3x}{sh(2x-5)}.$

24. $y=\dfrac{\arccos^3 5x}{th(x-2)}.$

25. $y=\dfrac{\sqrt{\arccos 3x}}{sh^2x}.$

26. $y=\dfrac{\arcsin^2 3x}{\sqrt{thx}}.$

27. $y=\dfrac{arctg^2 5x}{\sqrt[3]{cthx}}.$

28. $y=\dfrac{arctg^2 5x}{th(x+3)}.$

29. $y=\dfrac{\sqrt{sh^3x}}{arctg5x}.$

30. $y=\dfrac{\sqrt[5]{ch3x}}{arctg(x+2)}.$

10– TOPSHIRIQ

1. $y=\dfrac{9arctg(x+7)}{(x-1)^2}.$

2. $y=\dfrac{8arctg(2x+3)}{(x+1)^3}.$

3. $y=\dfrac{7\arccos(4x-1)}{(x+2)^4}.$

4. $y=\dfrac{6\arcsin(x+5)}{(x-2)^5}.$

5. $y=\dfrac{3arcctg(2x-5)}{(x+1)^4}.$

6. $y=\dfrac{2arctg(3x+2)}{(x-3)^2}.$

7. $y=\dfrac{4\arccos 3x}{(x+2)^5}.$

8. $y=\dfrac{\arcsin(3x+8)}{(x-7)^3}.$

9. $y=\dfrac{7arctg(4x+1)}{(x-4)^2}.$

10. $y=\dfrac{3\arcsin(2x-7)}{(x+2)^4}.$

11. $y=\dfrac{2\lg(4x+5)}{(x+6)^4}.$

12. $y=\dfrac{5\ln(5x+7)}{(x-7)^2}.$

13. $y=\dfrac{4\log_3(3x+1)}{(x+1)^2}.$

14. $y=\dfrac{7\log_4(2x-5)}{(x-1)^5}.$

15. $y=\dfrac{\ln(7x+2)}{(x-6)^4}.$

16. $y=\frac{4\lg(3x+7)}{(x+1)^7}.$

17. $y=\frac{5\log_2(x^2+1)}{(x-3)^4}.$

18. $y=\frac{6\log_3(2x+9)}{(x+4)^2}.$

19. $y=\frac{3\log_2(5x-4)}{(x-3)^5}.$

20. $y=\frac{7\log_5(x^2+x)}{(x+3)^3}.$

21. $y=\frac{\log_7(2x^2+5)}{(x-4)^2}.$

22. $y=\frac{2\ln(3x-10)}{(x+5)^7}.$

23. $y=\frac{8\lg(4x+5)}{(x-1)^5}.$

24. $y=\frac{2\log_3(4x-7)}{(x+3)^4}.$

25. $y=\frac{3\log_4(2x+9)}{(x-7)^2}.$

26. $y=\frac{\lg(x^2+2x)}{(x+8)^4}.$

27. $y=\frac{3\ln(x^2+5)}{(x-7)^3}.$

28. $y=\frac{4\log_2(3x-5)}{(x-2)^2}.$

29. $y=\frac{2\ln(2x^2+3)}{(x-7)^4}.$

30. $y=\frac{4\lg(3x+7)}{(x-5)^3}.$

11– TOPSHIRIQ

1. $y=\sqrt{\frac{2x+1}{2x-1}}\log_2(x-3x^2).$

2. $y=\sqrt[3]{\frac{2x-5}{2x+3}}\lg(4x+7).$

3. $y=\sqrt[4]{\frac{x+3}{x-3}}\ln(5x^2-2x+1).$

4. $y=\sqrt[5]{\frac{x+1}{x-1}}\log_3(x^2+x+4).$

5. $y=\sqrt[6]{\frac{7x-4}{7x+4}}\log_5(3x^2+2x).$

6. $y=\sqrt[7]{\frac{2x-3}{2x+1}}\lg(7x-10).$

7. $y=\sqrt[8]{\frac{5x+1}{5x-1}}\ln(3x-x^2).$

8. $y=\sqrt[9]{\frac{x+3}{x-3}}\log_5(2x-3).$

9. $y=\sqrt{\frac{6x+5}{6x-5}}\lg(4x+7).$

10. $y=\sqrt[3]{\frac{4x-1}{4x+1}}\ln(2x^3-3).$

11. $y=\sqrt[4]{\frac{x+6}{x-6}}\sin(3x^2+1).$

12. $y=\sqrt[5]{\frac{x-7}{x+7}}\cos(2x^3+x).$

13. $y=\sqrt[6]{\frac{x-9}{x+9}}tg(3x^2-4x+1).$

14. $y=\sqrt[7]{\frac{x-4}{x+4}}ctg(2x+5).$

15. $y=\sqrt[8]{\frac{x-2}{x+2}}\sin(4x^2-7x+2).$

16. $y=\sqrt[9]{\frac{x-3}{x+3}}\cos(x^2-3x+2).$

17. $y=\sqrt{\frac{3x-2}{3x+2}}tg(2x^2-9).$

18. $y=\sqrt{\frac{2x+3}{2x-3}}ctg(3x^2+5).$

19. $y=\sqrt[4]{\frac{x+5}{x-5}}\sin(3x^2-x+4).$

20. $y=\sqrt[5]{\frac{x-6}{x+6}}\cos(7x+2).$

21. $y=\sqrt[6]{\frac{x-7}{x+7}}\arcsin(2x+3).$

22. $y=\sqrt[7]{\frac{x-8}{x+8}}\arccos(3x-5).$

23. $y=\sqrt[8]{\frac{x-4}{x+4}}arctg(5x+1).$

24. $y=\sqrt[9]{\frac{x-1}{x+1}}arcctg(7x+2).$

25. $y=\sqrt{\frac{7x-4}{7x+4}}\arcsin(x^2+1).$

26. $y=\sqrt[3]{\frac{8x-3}{8x+3}}\arccos(x^2-5).$

27. $y=\sqrt[4]{\frac{2x-5}{2x+5}}arctg(3x+2).$

28. $y=\sqrt[5]{\frac{3x-4}{3x+4}}arcctg(2x+5)$ **.**

29. $y=\sqrt[6]{\frac{x^2-1}{x^2+1}}\arcsin 2x.$

30. $y=\sqrt[7]{\frac{x^2+3}{x^2-3}}\arccos 4x.$

12– TOPSHIRIQ

1. $y=(cth3x)^{\arcsin x}.$

2. $y=(\cos(x+2))^{\ln x}.$

3. $y=(\sin 3x)^{\arccos x}.$

4. $y=(th5x)^{\arcsin(x+1)}.$

5. $y=(sh(x+2))^{\arcsin 2x}.$

6. $y=(\cos 5x)^{arctg\sqrt{x}}.$

7. $y=(\sqrt{3x+2})^{arcctg3x}.$

8. $y=(\ln(x+3))^{\sin\sqrt{x}}.$

9. $y=(\log_2(x+4))^{ctg7x}.$

10. $y=(sh3x)^{arctg(x+2)}.$

11. $y=(ch3x)^{ctg\frac{1}{x}}.$

12. $y=(\arcsin 5x)^{tg\sqrt{x}}.$

13. $y=(\arccos 5x)^{\ln x}.$

14. $y=(arctg 2x)^{\sin x}.$

15. $y=(\ln(x+7))^{ctg 2x}.$

16. $y=(ctg(7x+4))^{\sqrt{x+3}}.$

17. $y=(th\sqrt{x+1})^{arctg2x}.$

18. $y=(cth\frac{1}{x})^{\arcsin 7x}.$

19. $y=(\cos(x+5))^{\arcsin 3x}.$

20. $y=(\sqrt{x+5})^{\arccos 3x}.$

21. $y=(\sin 4x)^{arctg\frac{1}{x}}.$

22. $y=(tg3x^4)^{\sqrt{x+3}}.$

23. $y=(ctg 2x^3)^{\sin\sqrt{x}}.$

24. $y=(tg7x^5)^{\sqrt{x+2}}.$

25. $y=(\arccos x)^{\sqrt{\cos x}}.$

26. $y=(ctg\,7x)^{sh(x+3)}$.

27. $y=(sh5x)^{arctg(x+2)}$.

28. $y=(arctgx)^{th(3x+1)}$.

29. $y=(cth\sqrt{x})^{\sin(x+3)}$.

30. $y=(sh3x)^{arcctg2x}$.

13– TOPSHIRIQ

1. $y=(\arccos(x+2))^{tg3x}$.

2. $y=(\arcsin 2x)^{ctg(x+1)}$.

3. $y=(arctg(x+7))^{\cos 2x}$.

4. $y=(arcctg(x-3))^{\sin 4x}$.

5. $y=(ctg(3x-2))^{\arcsin 3x}$.

6. $y=(tg(4x-3))^{\arccos 2x}$.

7. $y=(\cos(2x-5))^{arctg5x}$.

8. $y=(\sin(7x+4))^{arcctgx}$.

9. $y=(\arcsin 2x)^{\ln(x+3)}$.

10. $y=(\arccos 3x)^{\lg(5x-1)}$.

11. $y=(arctg5x)^{\log_2(x+4)}$.

12. $y=(arctg7x)^{\lg(x+1)}$.

13. $y=(\log_4(2x+3))^{\arcsin x}$.

14. $y=(\log_5(3x+2))^{\arccos x}$.

15. $y=(\lg(7x-5))^{arctg2x}$.

16. $y=(\ln(5x-4))^{arcctgx}$.

17. $y=(\log_2(6x+5))^{\arcsin 2x}$.

18. $y=(\lg(4x-3))^{\arccos 4x}$.

19. $y=(\ln(7x-3))^{arctg5x}$.

20. $y=(\log_5(2x+5))^{arctgx}$.

21. $y=(\sin(8x-7))^{cth(x+3)}$.

22. $y=(\cos(3x+8))^{th(x-7)}$.

23. $y=(tg(9x+2))^{ch(2x-1)}$.

24. $y=(ctg(7x+5))^{sh3x}$.

25. $y=(sh(3x-7))^{\cos(x+4)}$.

26. $y=(ch(2x-3))^{tg(x+5)}$.

27. $y=(th(7x-5))^{\sin(x+2)}$.

28. $y=(ch(3x+2))^{\cos(x+4)}$.

29. $y=(\ln(7x+4))^{tgx}$.

30. $y=(\lg(8x+3))^{tg5x}$.

14– TOPSHIRIQ

1. $y=\dfrac{\sqrt{x+7}(x-3)^4}{(x+2)^5}$.

2. $y=\dfrac{(x-3)^5(x+2)^3}{\sqrt{(x-1)^3}}$.

3. $y=\dfrac{(x-2)^3\sqrt{(x+1)^5}}{(x-4)^2}$.

4. $y=\dfrac{(x+3)\sqrt[5]{(x-2)^2}}{(x+1)^7}$.

5. $y=\dfrac{(x+2)^7(x-3)^3}{\sqrt{(x+1)^5}}$.

6. $y=\dfrac{(x-1)^4(x+2)^5}{\sqrt[3]{(x-4)^2}}$.

7. $y=\dfrac{(x-3)^2\sqrt{x+4}}{(x+2)^7}.$

8. $y=\dfrac{(x-7)^{10}\sqrt{3x-1}}{(x+3)^5}.$

9. $y=\dfrac{(x+1)^8(x-3)^2}{\sqrt{(x+2)^5}}.$

10. $y=\dfrac{(x+2)(x-7)^4}{\sqrt[3]{(x-1)^4}}.$

11. $y=\dfrac{\sqrt[5]{(x+4)^3}}{(x-1)^2(x+3)^5}.$

12. $y=\dfrac{\sqrt[3]{(x-1)^7}}{(x+1)^5(x-5)^3}.$

13. $y=\dfrac{\sqrt{(x+2)^3}(x-1)^4}{(x+2)^7}.$

14. $y=\dfrac{\sqrt[3]{(x-2)^5}(x+3)^2}{(x-7)^3}.$

15. $y=\dfrac{\sqrt[4]{x-8}(x+2)^6}{(x-1)^5}.$

16. $y=\dfrac{\sqrt[5]{x+1}(x-3)^7}{(x+8)^3}.$

17. $y=\dfrac{\sqrt[7]{(x-2)^4}}{(x+1)^2(x-6)^5}.$

18. $y=\dfrac{\sqrt[5]{(x+1)^2}}{(x-3)^4(x-4)^3}.$

19. $y=\dfrac{\sqrt{x^2+2x-3}}{(x+3)^7(x-4)^2}.$

20. $y=\dfrac{\sqrt[3]{(x-2)^4}}{(x-5)(x+1)^7}.$

21. $y=\dfrac{(x+4)^3(x-2)^3}{\sqrt[5]{(x-2)^2}}.$

22. $y=\dfrac{(x-1)^6(x+2)^3}{\sqrt[5]{(x+3)^2}}.$

23. $y=\dfrac{(x-1)^4(x-7)^2}{\sqrt[3]{(x+2)^5}}.$

24. $y=\dfrac{(x+7)^2(x-3)^5}{\sqrt{x^2+3x-1}}.$

25. $y=\dfrac{\sqrt[3]{x-3}(x+7)^5}{(x-4)^2}.$

26. $y=\dfrac{\sqrt{x+10}(x-8)^3}{(x-1)^5}.$

27. $y=\dfrac{\sqrt[5]{(x-2)^3}(x-1)}{(x+3)^4}.$

28. $y=\dfrac{\sqrt[4]{(x+1)^3}(x-2)^5}{(x-3)^2}.$

29. $y=\dfrac{\sqrt[6]{(x-1)^5}}{(x+2)^4(x-5)^7}.$

$y=\dfrac{\sqrt[5]{(x+2)^3}}{(x-1)^4(x-3)^5}.$

NAMUNAVIY VARIANTNI YECHISH

Berilgan funksiyalarni differensiyalang

1. $y=9x^5-\frac{4}{x^3}+\sqrt[3]{x^7}-3x+4.$

Yechish: $y'=9\cdot 5x^4-4(-3)x^{-4}+\frac{7}{3}x^{\frac{4}{3}}-3=45x^4+\frac{12}{x^4}+\frac{7}{3}\sqrt[3]{x^4}-3.$

2. $y=\sqrt[4]{(2x^2-3x+1)^3}-\frac{6}{(x+1)^3}.$

Yechish: $y'=\frac{3}{4}(2x^2-3x+1)^{-\frac{1}{4}}(4x-3)-6(-3)(x+1)^{-4}=\frac{3}{4}\frac{4x-3}{\sqrt[4]{2x^2-3x+1}}+\frac{18}{(x+1)^4}.$

3. $y=tg^5(x+2)\cdot\arccos 3x^2.$

Yechish: $y'=5tg^4(x+2)\cdot\frac{1}{\cos^2(x+2)}\arccos 3x^2+tg^5(x+2)\cdot\left(\frac{1}{\sqrt{1-9x^4}}\right)\cdot 6x=$

.

$=\frac{5tg^4(x+2)\cdot\arccos 3x^2}{\cos^2(x+2)}-\frac{tg^5(x+2)\cdot 6x}{\sqrt{1-9x^4}}$

4. $y=\arcsin^5 4x\cdot\log_2(x-5).$

Yechish $y'=5\arcsin^4 4x\cdot\frac{1}{\sqrt{1-16x^2}}\cdot 4\log_2(x-5)+\arcsin^5 4x\cdot\frac{1}{(x-5)\ln 2}=$

$=\frac{20\arcsin^4 4x\cdot\log_2(x-5)}{\sqrt{1-16x^2}}+\frac{\arcsin^5 4x}{(x-5)\ln 2}.$

5. $y=3^{-x^4}ctg\,7x^3.$

Yechish:

$y'=3^{-x^4}\ln 3\cdot(-4x^3)ctg\,7x^3+3^{-x^4}(\frac{1}{-\sin^2 7x^3})\cdot 21x^2=-4\ln 3\cdot 3^{-x^4}ctg\,7x^3-\frac{21x^3\cdot 3^{-x^4}}{\sin^2 7x^3}.$

6. $y=cth^2 3x\cdot arctg\sqrt{x}.$

Yechish: $y'=2cth3x\cdot\left(-\frac{1}{sh^2 3x}\right)\cdot 3arctg\sqrt{x}+cth^2 3x\times\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}=$

$=-\frac{6cth3x\cdot arctg\sqrt{x}}{sh^2 3x}+\frac{cth^2 3x}{(1+x)2\sqrt{x}}.$

7. $y=\sqrt{3x^2-7x+\frac{5}{e^{-x^4}}}.$

Yechish: $y'=(\sqrt{3x^2-7x+\frac{5}{e^{-x^4}}}.)'=\frac{(6x-7)e^{x^4}}{2\sqrt{3x^2-7x+5}}+\sqrt{3x^2-7x+5}\cdot e^{x^4}\cdot 4x^3=$

$$=\frac{(6x-7)e^{x^4}}{2\sqrt{3x^2-7x+5}}+4x^3e^{x^4}\sqrt{3x^2-7x+5}.$$

8 $y=\frac{(\lg(x^2-3x+5))}{arcctg^2 5x}.$

Yechish:

$$y'=\left(\frac{2x-3}{(x^2-3x+5)\ln 10}arcctg^2 5x-\lg(x^2-3x+5)\times 2\cdot arcctg 5x\cdot\left(-\frac{1}{1+25x^2}\right)\cdot 5\right)*$$

$$*\,arcctg^4 5x=\left(\frac{(2x-3)arcctg^2 5x}{(x^2-3x+5)\ln 10}+\frac{10\lg(x^2-3x+5)\cdot arcctg 5x}{1+25x^2}\right)\cdot arcctg^{-4} 5x.$$

9. $y=\sqrt{\frac{\arcsin 3x}{sh^2 x}}.$

Yechish: $y'=\dfrac{\frac{1}{2\sqrt{\arcsin 3x}}\frac{1}{\sqrt{1-9x^2}}\cdot 3sh^2x-2shxchx\sqrt{\arcsin 3x}}{sh^4x}=.$

$$=\frac{\frac{3sh^2x}{2\sqrt{\arcsin 3x}\sqrt{1-9x^2}}-ch2x\sqrt{\arcsin 3x}}{sh^4x}$$

10. $y=\frac{(3\ln(x^2-5))}{(x+3)^7}.$

Yechish: $y'=3\dfrac{\frac{1}{x^2-5}\cdot 2x(x+3)^7-7(x+3)^6\ln(x^2-5)}{(x+3)^{14}}=$

$$=3\frac{\cdot\frac{2x(x+3)}{x^2-5}-7\cdot\ln(x^2-5)}{(x+3)^8}$$

11. $y=\sqrt[7]{\frac{(x+5)}{(x-5)}}ctg(3x-4).$

Yechish: $y'=\frac{1}{7}\left(\frac{x+5}{x-5}\right)^{-\frac{6}{7}}\frac{x-5-(x+5)}{(x-5)^2}ctg(3x-4)-\frac{1}{\sin^2(3x-4)}\cdot 3\sqrt[7]{\frac{x+5}{x-5}}=$

$$=-\frac{10}{7}\frac{ctg(3x-4)}{\sqrt[7]{\frac{(x+5)^6}{(x-5)^6}}}-\frac{3}{\sin^2(3x-4)}\sqrt[7]{\frac{x+5}{x-5}}.$$

12. $y=(th\sqrt{x+2})^{\ln(3x+2)}$.

Yechish: Berilgan funksiyani logarifmlaymiz:

$\ln y=\ln(3x+2)\ln(th\sqrt{x+2}).$ Bu tenglikni differensiyalasak

$$\frac{1}{y}y'=\frac{3}{3x+2}\ln(th\sqrt{x+2})+\ln(3x+2)\cdot\frac{1}{th\sqrt{x+2}ch^2\sqrt{x+2}}\frac{1}{2\sqrt{x+2}}\cdot$$

Oxirgi tenglikdan y' ni topamiz

$$y'=(th\sqrt{x+2})^{\ln(3x+2)}\left(\frac{3\ln(th\sqrt{x+2})}{3x+2}+\frac{\ln(3x+2)}{2\sqrt{x+2}sh\sqrt{x+2}ch\sqrt{x+2}}\right).$$

13. $y=(\sin 7x)^{arctg(3x-5)}$.

Yechish: Berilgan funksiyani logarifmlaymiz: $\ln y=arctg(3x-5)\cdot\ln(\sin 7x)$

Bu tenglikni differensiyalasak,

$$\frac{1}{y}y'=\frac{3}{1+(3x-5)^2}\ln(\sin 7x)+7\frac{\cos 7x}{\sin 7x}arctg(3x-5)$$

Bu yerdan

$$y'=(\sin 7x)^{arctg(3x-5)}\cdot\left(\frac{3\cdot\ln(\sin 7x)}{1+(3x-5)^2}+7ctg\,7x\cdot arctg(3x-5)\right)$$

14. $y=\sqrt[7]{(x+5)^6}/\left((x-1)^2(x+3)^5\right)$.

Yechish: Logarifmik differensiyalash usulini qo‘llab ketma- ket topamiz:

$$\ln y=\frac{6}{7}\ln(x+5)-2\ln(x-1)-5\ln(x+3)$$

$$\frac{1}{y'}y=\frac{6}{7}\frac{1}{x+5}-2\frac{1}{x-1}-5\frac{1}{x+3}$$

$$y'=\frac{\sqrt[7]{(x+5)^6}}{(x-1)^2(x+3)^5}\left(\frac{6}{7(x+5)}-\frac{2}{x-1}-\frac{5}{x+3}\right)$$

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